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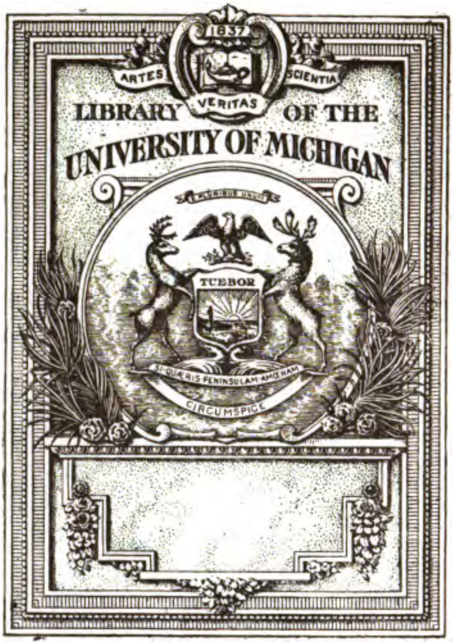
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ALTERNATING CURRENT MACHINERY

PART I.

INSTRUCTION PAPER

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ALTERNATING-CURRENT MACHINERY

PART I

INTRODUCTION

Relation of Power Supply to Industry. Before beginning the study of alternating currents and the description of the various types of alternating-current apparatus, it will be more interesting and profitable to get a bird's-eye view of the general plan of an alternating-current power system. But to fully grasp the plan of any particular power system, it is helpful to get first an appreciation of the relation of a power supply to the development of industry and to the promotion of the convenience and welfare of mankind.

One of the measures of the degree of civilization attained by a community, state, or nation is the extent and universality of its use of power. Among primitive peoples practically all work was done by hand. Manual labor was the rule; labor-saving machinery was all but unknown. Under modern conditions of civilization, however, some form or other of power is needed in almost every building, whether in town or country. Although the amount of power needed at any one place may be considered insignificant, yet the total amount used by a factory, a city block, or a community may be surprisingly large. Take, for example, a steel works. It needs power for a great many purposes: for hauling coal, coke, ore, limestone, and other raw materials; for driving rolling mills; and for operating elevators, hoists, fans, air compressors, and machine tools. It needs power for lighting offices, shops, and yards, and for operating various labor-saving devices too numerous to mention. The location of this host of power-consuming machines being widely scattered over different buildings and throughout different floors in any one building, it would, therefore, not be practicable or economical to install a source of power, like a steam or gas engine, at each point where power is wanted. The

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proper solution of the problem is to generate at one centrally located power station all the power needed for the entire plant, and then to distribute this power by suitable means to the various floors and buildings where it is to be utilized.

Creation and Utilization of Power. The power station contains the *prime movers*—such as steam engines or turbines, gas or oil engines, and water wheels which generate mechanical power—using as sources of energy coal, gas, oil, or water, as the case may be. In some small power plants and factories the prime movers still transmit their mechanical power through ropes or belts to a main line shaft equipped with pulleys which, by means of smaller belts, transmit the power to counter-shafts. By another series of pulleys and belts the counter-shaft is made to drive either individual machines or groups of machines. In the case of an electric power plant the prime movers are used to drive electric generators (dynamos), which generate electric power either in the form of direct currents or alternating currents. To prevent any possible misunderstanding, it should be emphasized that an electric generator is not of itself a source of power. It is a machine which merely serves to convert the energy of mechanical motion, imparted to it by a prime mover, into energy in the form of electric currents.

Tendency toward Large Power Plants. The same reasons, which, in the case of a mill, department store, or hotel, lead to a concentration of power generation at a power station for each, can in general be urged in favor of combining many individual isolated power plants into one or more large central power stations. Thus even in our largest cities, like New York, Chicago, Philadelphia, Boston, and many others, the great bulk of the power needed for municipal and private lighting, for manufacturing, traction, etc., is generated at a few huge central stations, some of them having a maximum output of over 100,000 kilowatts. The power so generated is distributed to consumers at a price so low that it oftentimes becomes cheaper to buy power than to generate it in small private plants. The tendency toward doing away with small uneconomical plants and buying power from the large central station is increasing. The economies possible in fuel consumed, in labor and materials costs, and in management, due to the use of large generating units operating under more favorable load conditions, are enormous and are not fully realized by the public.

Interlinking Power Lines. As the next logical step in the centralization of generating stations comes the plan recently proposed by the Secretary of the Interior—to link up existing central electric generating stations all along the Atlantic seaboard from Virginia up through the New England states by means of tie lines. The general scheme of interlinking the power lines of several electric-power companies has been for some years in operation on the Pacific coast and is highly successful. If this plan be carried out, as it should be, it would mean connecting together by copper feed wires the existing transmission lines of a large number of electric-power companies each operating now in a limited territory. The plan aims to create an almost continuous network of electric transmission wires paralleling the coast, making what has been called a "river of electric power."

The chief advantage of this proposal is that it would materially reduce the cost of electric power to the consumer, increase facilities for transportation, manufactures, and housekeeping, and greatly encourage and stimulate a more general use of electric power. It would mean saving of time and lessening of drudgery, and would add to the comfort and well-being of communities.

Methods of Transmitting Power. Having outlined the development of a power supply originally planned to meet the needs of a manufacturing firm or a group up to a general supply adequate for the needs of whole communities or even states, let us now consider the various methods available for transmitting power.

There are four common ways of transmitting power for considerable distances, as follows: (1) by mechanical means, using rope drives, belts, and shafting; (2) by compressed air conducted in pipes or flexible hose; (3) by steam under pressure flowing in pipes; (4) by electricity in the form of direct currents or alternating currents flowing in metallic conductors. Each of these methods has its peculiar advantages under certain limiting conditions, and where the distances involved are relatively short. When, however, the distance over which the power is to be transmitted becomes considerable, the first three methods become impracticable and prohibitively costly.

For a general system of power transmission electricity stands without a rival. The electric current passes over wires which are stationary, and which may be easily bent in turning corners and in avoiding obstructions. Wires are easily supported in various ways. Most important of all, the cost of electric transmission lines is relatively small, the power lost is small, and the depreciation and maintenance charges against the lines are also relatively small. Moreover, electric power may be turned on, shut off, and controlled by devices that are more positive in action, more rapid and satisfactory in their operation, than those which must be used when other methods of transmitting power are employed. The apparatus for the control and protection of electrical machinery is simpler, more compact, and more reliable on the whole than that required for other systems of power transmission. Electrical transmission over long distances is thus more economical, more flexible, more easily adapted to a variety of uses, and withal more convenient, than any other system.

Electric Power Systems. An electric power system involves three main features or processes: generation, transmission, and distribution, the utilization in each of which involves the use of either direct currents or alternating currents. There are in general, therefore, three kinds of electric power systems, as follows:

(1) A system in which direct-current power is generated, transmitted, and utilized.

(2) A system in which direct-current power is generated and transmitted, but is then converted into alternating current for use.

(3) A system in which alternating-current power is generated and transmitted, but utilized either in the form of alternating current, or in the form of converted direct current.

The earliest electric-power stations in this country were of the first type, and today some of the smaller stations are still furnishing direct current for incandescent and arc lighting, and for supplying direct-current motors over limited distances. The second system has not met with favor in the United States, although the Thury direct-current system has been in successful use for many years in European countries. The third system is almost exclusively used in this country today, most of the electric power used for lighting, industrial purposes, and traction being generated and transmitted in the form of alternating currents at high voltages. The alternating-current power delivered by the transmission lines to step-down, or reducing, transformers located at substations may then be distributed in the form of alternating currents, or may be converted into direct currents at voltages suited to those industries requiring it—such, for example, as electrochemical works and electric railways.

High Voltage Necessary. Long-distance electric power transmission to be economical or even commercially possible must be accomplished by using high voltages. Since electric power is the product of two factors—voltage and current—it follows that to be able to use small, inexpensive transmission wires, and to reduce the heating losses therein, the current should be reduced to a minimum. This for a given amount of power transmitted means using a voltage as high as practicable.

Advantages of Alternating Currents for Transmission. The transmission of direct currents on a large scale at high voltages has so many disadvantages that it has never been adopted in this country. The principal advantages of alternating currents for long-distance power transmission are as follows:

1. The generators for alternating currents, called *alternators*, can be built in larger sizes and more cheaply than those for direct currents. Thus alternators driven by steam turbines have been built to deliver 60,000 kilowatts, while the largest direct-current generators are limited to an output of about 2500 kilowatts and are inherently not adapted for high speeds.

2. Alternating-current power can more easily and cheaply be converted into power at either a higher or a lower voltage through the agency of a very simple, cheap, and efficient apparatus—the transformer. Transformers cannot be used on direct-current circuits.

3. The alternating-current system is much more flexible, more easily controlled, and more adaptable to various kinds of service.

4. By combining two or three alternating-current circuits into a polyphase (a two- or a three-phase) system it becomes possible to convert electrical into mechanical power through the use of motors of simple, compact, and rugged construction, such as induction and synchronous motors.

Physical Nature of Alternating Current. The flow of electricity in the form of an electric current, may take place in any one of three ways:

(1) The flow may be continuous, never changing in value or direction, in which case it is called a steady direct current.

(2) The flow may always be in the same direction, but its rate of flow may change, in which case it is called a pulsating direct current.

(3) The flow may change in both magnitude and direction in a periodic manner, in which case it is called an alternating current.

Steady Direct Current. The first case, that of a steady direct flow, is analogous to the flow of water in a river, or the flow of water in a pipe connected to a centrifugal water pump. In either of these cases the water flows steadily in the same direction.

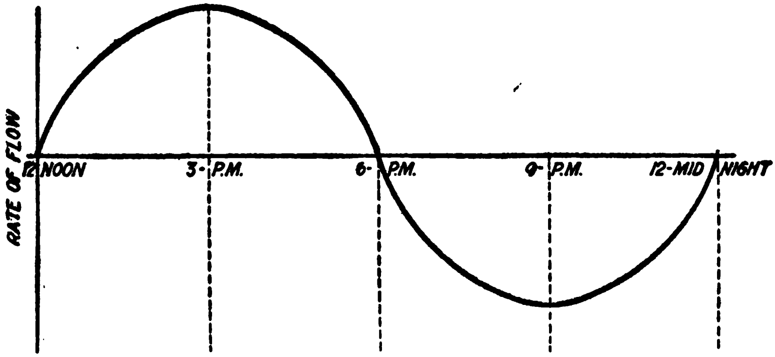

Fig. 1. Curve Showing Rate of Flow of Tide

Pulsating Direct Current. The second case, that of a pulsating current, is analogous to the flow of blood through the arteries. Here the pumping action of the heart causes the flow of blood to vary periodically in value, but the direction of flow is never reversed. In a similar manner a piston type of pump forces water through a pipe; the flow is pulsating but always in the same direction.

Periodic Alternating Current. The third type of flow, the alternating flow, is analogous to the coming in and going out of tides. In this case there is a distinct and periodic reversal in direction of flow.

Analogy of Tides. Suppose that the tide is lowest at 12 o'clock noon, Fig. 1. At this instant the water is neither going out nor coming in, that is, its rate of flow is zero; an instant later the water begins to flow in, and the rate at which it flows in continually increases until 3 P.M. After 3 P.M. the water continues to flow in but at a lesser rate until 6 P.M., when the tide is highest. But at this instant the water is neither flowing in nor out; that is, its rate of flow is again zero.

The curve, Fig. 1, is plotted to show how the rate of flow, not the height of the tide, changes during the day or with time. After 6 P.M. the water begins to flow out at an increasing rate until 9 P.M., when it is flowing out at its greatest rate. From 9 P.M. to 12 midnight the water continues to flow out, but at a decreasing rate. At 12 midnight the water has ceased completely to flow out; the tide is at its lowest ebb. At this point, therefore, the tide has gone through a complete cycle and is ready to start a new cycle of movements in endless repetition.

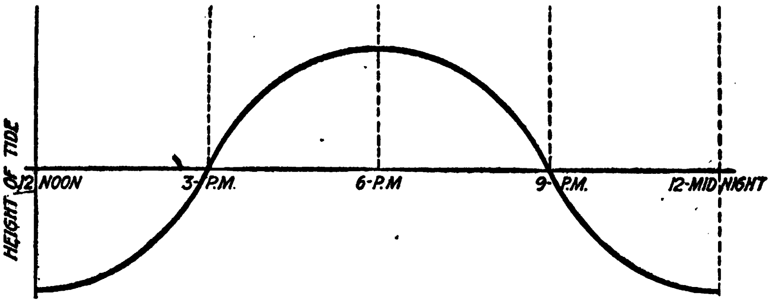

Fig. 2. Curve Showing Height of Tide

The time taken to complete the full cycle was 12 hours, which is called the periodic time, or simply the period. Thus the tide goes through about two cycles per day of 24 hours. The number of cycles, two in this case, completed in a unit of time would be called the frequency.

In a similar way an alternating current periodically flows back and forth in a circuit, but the time it takes to go through a complete set of positive and negative values, that is, its period, is much less than for the tidal flow. The number of cycles through which the current passes in the same unit of time is therefore very much larger. The frequency of an alternating current is expressed in cycles per second.

Phase Relation. If, as in Fig. 2, we plot the height of the water or tide for the different values of time taken in Fig. 1, we find that the height of the tide likewise goes through a complete cycle in the same time as was taken by the flow of water. It should be observed, however, that when the tide is lowest the water is not moving at all, and likewise when the tide is highest the rate of flow is zero. In alternating-current terms it would be said that the height of the tide is *out of phase* with the rate of flow of water. In this case, calling one cycle 360 degrees, it is out of phase by one quarter of a cycle, or 90 degrees.

ALTERNATING ELECTROMOTIVE FORCES AND CURRENTS

Simple Alternator. The alternator is an arrangement by means of which mechanical energy is used to cause the magnetic flux from a magnet to pass through the opening of a coil of wire first in one and then in the opposite direction. This varying magnetic flux induces in the coil, first in one direction and then in the other, what is called an *alternating electromotive force,* which in turn produces an *alternating current* in the coil, and in the circuit which is connected to the terminals of the coil.

In the common type of alternator, the above-mentioned magnet and coil move relatively to each other. Fig. 3 shows the essential features of such an alternator. The poles *N*, *S*, *N*, *S*, etc., of a multipolar magnet called the *field magnet,* project radially inward toward the passing teeth *a a a* of a rotating mass *A* of laminated iron; and upon these teeth are wound coils of wire *c c*, in which the alternating electromotive force is induced. The rotating mass of iron with its windings of wire is called the *armature.* At one end of the armature (not shown in the figure) are mounted two insulated metal rings *r r*, called *collecting rings.* These metal rings are connected to the ends of the armature winding, and metal brushes *b b* rub on these rings, thus keeping the ends of the armature winding in continuous contact with the terminals of the external circuit to which the alternator supplies alternating current. No external circuit is shown in the figure.

The electromotive forces induced in adjacent armature coils are in opposite directions at each instant, and the coils are so connected together that these electromotive forces do not oppose each other. This is done by reversing the connections of every alternate coil, as indicated by the dotted lines connecting the coils in Fig. 3. The electromagnetic action of this type of alternator depends only upon the *relative* motion of field magnet and armature, and large machines are usually built with stationary armature and revolving field magnet. In the type of machine illustrated in Fig. 3, the armature revolves while the field magnet is stationary. This type, called the *revolving-armature* type, is generally adopted in small alternators.

The field magnet of an alternator is usually an electromagnet which is excited by a continuous electric current supplied by an independent generator, generally by an auxiliary continuous-current dynamo, called the *exciter*. The exciting current flows through coils of wire wound on the projecting poles *N*, *S*, *N*, *S* of the field magnet. These coils are not shown in Fig. 3.

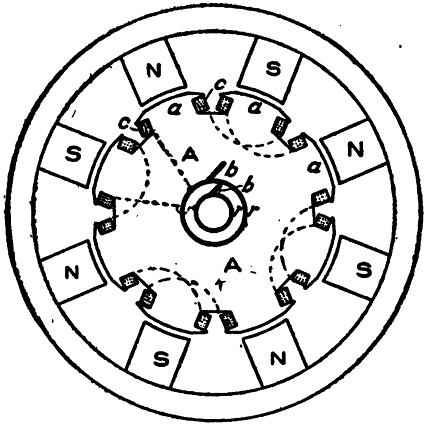

Fig. 3. Diagram of Armature and Field of Simple Alternator

The type of *armature core* shown in Fig. 3 is called the *toothed* armature core; and the *armature winding* is said to be *concentrated*, that is, the armature conductors are grouped in a few heavy bunches. Armature cores are also made with many small slots, in which the armature conductors are grouped in small bunches. This type of core is called a *multi-toothed* core, and the winding is said to be *distributed*.

In some of the earlier types of alternators the armature core consisted of a smooth, cylindrical mass of laminated iron, upon the face of which the conductors were arranged in bands side by side, one layer or more in depth. This type of armature is called the *smooth-core* armature; it has been superseded by the toothed core type.

Variations of Electromotive Force. *Cycle.* The electromotive force of an alternator passes through a set of *positive* values while a given coil of the armature is passing from a south to a north pole of the field magnet, and through a similar set of *negative* values while the coil is passing from a north to a south pole. The complete set of values, including positive and negative is called a *cycle.*

Frequency. Frequency is equal to the number of cycles per second; it is sometimes expressed by stating the number of alternations or reversals per minute. For example, an alternator having a frequency of 133 cycles per second has 266 reversals or alternations per second, or 15,960 alternations per minute. Frequencies are sometimes specified in alternations per minute, but specification in cycles per second is the more usual practice and is preferable.

$$\text{cycles per sec.} = \frac{\text{alternations per min.}}{2 \times 60}$$

Period. The fractional part of a second occupied by one cycle is called the periodic time, or period, of the alternating electromotive force or current.

Let f be the frequency in cycles per second, and T the period expressed as a fraction of a second. Then

$$f = \frac{1}{T} \qquad (1)$$

Therefore, if an alternating current has a frequency of 60 cycles per second, the period T of one cycle is one-sixtieth of a second.

Relations Between Speed and Frequency. Let p be the number of poles of the field magnet of an alternating-current machine; let n be the speed of its armature in revolutions per minute; and let f be the frequency of its electromotive force in cycles per second. Then

$$f = \frac{p}{2} \times \frac{n}{60} \qquad (2)$$

Examples. 1. A certain alternator has 10 poles, and runs at 1,500 revolutions per minute. What is its frequency?

SOLUTION. Substituting 10 for p, and 1,500 for n, in equation (2), we have

$$f=\frac{10}{2}\times\frac{1500}{60}=125 \text{ cycles per second}$$

2. An alternator is to run at 600 revolutions per minute and is to give a frequency of 60 cycles per second. What number of poles is required?

SOLUTION. Solving equation (2) for p, we have

$$p=\frac{2\times 60\times f}{n}$$

from which, substituting $f=60$, and $n=600$, we have

$$p=\frac{2\times 60\times 60}{600}=12 \text{ poles}$$

Advantages and Disadvantages of Alternating Currents. The electric transmission of a given amount of power may be accomplished by a large current at low electromotive force, or by a small current at high electromotive force. In the first case very large and expensive transmission wires must be used, or the loss of power in the transmission line will be excessive. In the second case comparatively small and inexpensive transmission wires may be used. Thus it is a practical necessity to employ high electromotive forces in long-distance transmission of power.

Example. It is desired to transmit 1,000 kilowatts of power over a distance of 10 miles, supposing that a loss in the line of 10 per cent of the power delivered is considered permissible. This corresponds to a loss of 100 kilowatts.

SOLUTION.—*Case 1.* Suppose that the electromotive force at the receiving end of the line is to be 100 volts. Then the current would be 1,000,000 watts divided by 100 volts, or 10,000 amperes. The resistance of the line must be such that the watts lost in the line—namely 100,000 watts—would be equal to I^2R, so that the resistance R of the line must be $\frac{W}{I^2}$, or 0.001 ohm. This would require two transmission wires each 10 miles long and 33 inches in diameter, or a total weight of 175,000 tons of copper, which would cost about $52,500,000.

Case 2. Suppose that the electromotive force at the receiving end of the line is to be 1,000 volts. Then the current would be 1,000,000 watts divided by 1,000 volts, or 1,000 amperes. The resistance of the line must be such that the watts lost in the line—namely, 100,000 watts—would be equal to I^2R, so that the resistance R of the line must be 0.1 ohm. This would require two transmission wires, each 10 miles long and 3.3 inches in diameter, or a total weight of 1,750 tons of copper, which would cost about $525,000.

TABLE I

Size and Cost of Copper Wire—Two-Wire System

To transmit 1,000 kilowatts a distance of 10 miles (one way) with a line loss equal to 10 per cent of the power delivered, for three different values of electromotive force

Volts at Receiving End of Line E	Amperes in Line I	Ohms in Line R	Diameter of Wire, in Inches	Weight of Wire, in Tons	Cost of Line Copper, in Dollars
100	10,000	0.001	33	175,000	52,500,000
1,000	1,000	0.1	3.3	1,750	525,000
10,000	100	10.0	0.33	17.5	5,250

Case 3. Suppose that the electromotive force at the receiving end of the line is to be 10,000 volts. Then the current would be 1,000,000 watts divided by 10,000 volts, or 100 amperes. The resistance of the line must be such that the watts lost in the line—namely 100,000 watts—would be equal to I^2R, so that the resistance R of the line must be 10 ohms. This would require two transmission wires, each 10 miles long and 0.33 inch in diameter, or a total weight of 17.5 tons of copper, which would cost about $5,250.

These results are summarized in Table I.

Transformation of High E. M. F.'s. High electromotive forces are dangerous under the conditions that ordinarily obtain among users of electric light and power; and many types of apparatus, such as incandescent lamps, operate satisfactorily only with medium or low electromotive forces. Therefore, means must be provided, at a receiving station, for transforming the power delivered, from high electromotive force and small current to low electromotive force and large current, if long-distance transmission is to be successful. This is called *step-down transformation.* The advantage of the alternating current over the direct current lies almost wholly in the cheapness of construction and of operation, and in the high efficiency of the alternating-current apparatus as compared with the direct-current apparatus that is required for transformation.

In step-down transformation of direct current, a motor takes a small current from the high-electromotive-force transmission mains, and drives a dynamo which delivers large current to service mains at low electromotive force. This apparatus, or its equivalent, the *dynamotor,* is expensive to construct; it requires attention in operation; and its efficiency is never, perhaps, above 90 per cent.

The step-down transformation of alternating currents is accomplished by means of the *alternating-current transformer,* which is described later on. The alternating-current transformer is very much cheaper than a dynamo and motor of the same output; it requires no attention in operation; and its efficiency under full load is usually greater than 97 per cent, especially in large sizes.

Simple Construction of A. C. Machines. The alternating current has some minor advantages over the direct current on account of the fact that alternating-current machines are frequently simpler in construction than direct-current machines. In particular, the commutator is not an essential part of an alternating-current generator. Again, in the case of the inductor alternator and the induction motor, the rotating part may not have any sliding electrical contacts whatever.

Miscellaneous A. C. Machines. The simple *single-phase* alternating current is not well adapted to general power service. The single-phase alternating-current induction motor does not start satisfactorily under load, in the case of large machines, although self-starting single-phase motors up to perhaps 20 horse-power are in commercial use, where neither direct-current nor polyphase alternating-current machines are available.

The single-phase series commutator motor within a few years has been developed especially for electric railway service both for trolley cars and for electric locomotives. Its operating characteristics, resembling closely those of the direct-current series motor, however, are not suitable for general power requirements. This type of motor is used on the electric locomotives of the New York, New Haven and Hartford Railroad and others.

For uninterrupted service the *synchronous motor* is frequently used, the starting being effected by an auxiliary engine or other independent mover. The synchronous motor is not satisfactory when frequent starting is necessary, for such service the *induction motor* being used. The simple induction motor, to start satisfactorily, must be supplied with two or more distinct alternating currents transmitted to the motor over separate lines. This is called the *polyphase* system of transmission, and is reserved for full treatment in later pages.

For some purposes, especially for the electrolytic processes used on a large scale in electro-chemical works, only direct current can be used. When power transmitted by alternating current is to be delivered in the form of direct current, the conversion is effected by means of the *rotary converter*, *motor generator*, or *mercury rectifier*.

Comparison of Direct- and Alternating-Current Problems. *Direct Current.* In direct-current work the electrical engineer is concerned with the relations between electromotive force, resistance, current, and power. These relations are determined by the applications of the following laws:

POWER LAW for direct-current circuits

$$P = EI \qquad (3)$$

in which P is the total power in watts delivered to a circuit by a generator of which the terminal electromotive force is E volts, when it produces a current of I amperes.

OHM'S LAW for direct-current circuits

$$I = \frac{E}{R} \qquad (4)$$

in which I amperes is the steady current produced by E volts acting on a circuit of R ohms resistance.

JOULE'S LAW for direct-current circuits

$$P = I^2R \qquad (5)$$

in which P is the power in watts expended in heating a circuit of R ohms resistance, when a current of I amperes is forced through the circuit.

KIRCHOFF'S LAWS for direct-current circuits.

1. When a circuit branches, the current in the main circuit is equal to the sum of the currents in the separate branches.

2. (a) When two or more sources of electromotive force are connected in series, the total electromotive force is the sum of the individual electromotive forces.

3. (b) When an electromotive force acts on a number of elements or things in series, it is subdivided into parts, each of which acts upon one of the elements, and the sum of these parts is equal to the total electromotive force. For example, an arc-light dynamo of which the terminal electromotive force is 3,000 volts,

acts on 60 similar arc lamps connected in series. Neglecting the resistance of the connecting wires, each lamp is acted upon by one-sixtieth of the total electromotive force, or by 50 volts.

Alternating Current. In alternating-current work the electrical engineer is likewise concerned with the relations between electromotive force, resistance, current, and power. These relations are determined by the application of the same fundamental laws as in the case of direct currents, but in more or less modified forms.* A summary of the fundamental laws of alternating currents is here given simply for purposes of comparison.

POWER LAW for alternating-current circuits

$$P = EI \cos \theta \qquad (6)$$

in which P is the power in watts delivered to a circuit by an alternator of which the "effective" terminal electromotive force is E volts, when it produces an "effective" current of I amperes in a circuit, and $\cos \theta$ is what is called the "power factor" of the circuit.

OHM'S LAW for alternating-current circuits

$$I = \frac{E}{\sqrt{R^2 + X^2}} \qquad (7)$$

in which I is the "effective" current in amperes produced by an "effective" electromotive force of E volts acting on a circuit of which the resistance is R ohms, and the "reactance" is X ohms. The expression $\sqrt{R^2+X^2}$ in equation (7) is called the *impedance*, and it is expressed in ohms.

JOULE'S LAW for alternating-current circuits

$$P = I^2 R \qquad (5)$$

in which P is the power in watts expended in heating a circuit of R ohms resistance when an "effective" alternating current of I amperes is forced through the circuit.

KIRCHOFF'S LAWS for alternating-current circuits

1. When an alternating-current circuit branches, the "effective" current in the main circuit is the "geometric"† (or "vector") sum of the "effective" currents in the separate branches.

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*The student is not expected to understand fully the reasons for the statements here given, until he has completed Parts I and II.

†Alternating electromotive forces and alternating currents are added in the same way that forces are added, that is, by means of the principle known as the "parallelogram of forces."

Example. An alternator A, Fig. 4, supplies an effective current of I amperes in the main circuit, which divides into two branches. The effective currents in the two branches are I_1 and I_2 amperes, respectively. The relation between I, I_1, and I_2 is shown in Fig. 5. The angles θ_1 and θ_2 depend upon the relative values of the resistance and reactance of the respective branches, as is explained later. It is to be particularly noticed that the arithmetical sum of I_1 and I_2 is in general greater than I.

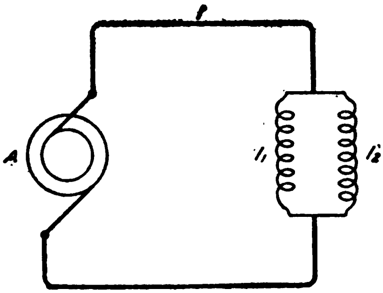


Fig. 4. Diagram of a Branched Alternating Circuit

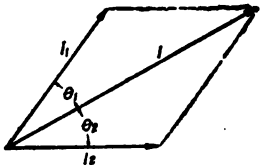


Fig. 5. Vector Diagram of Branched Circuit

2. (a) When two or more alternators (or transformer secondaries) are connected in series, the total effective electromotive force is the "geometric" (or "vector") sum of the effective electromotive forces of the individual alternators.

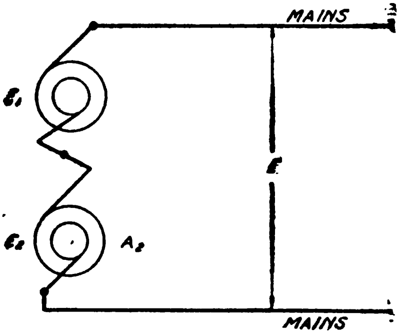


Fig. 6. Two Alternators in Series

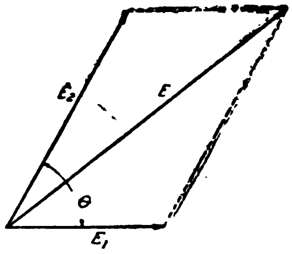


Fig. 7. Vector Diagram of E. M. F.s for Two Alternators in Series

Example. Two alternators A_1 and A_2, Fig. 6, of which the effective electromotive forces are E_1 and E_2, respectively, are connected in series to supply mains. Then the effective electromotive force E, between mains, is the geometric sum of E_1 and E_2, as shown in Fig. 7. The angle θ depends upon the positions, relatively to the field magnets, of the armature coils on the respective machines.

(b) When an alternating electromotive force E acts upon a number of elements or things in series, it is subdivided into parts, each of which acts upon one of the elements, and the "geometric" (or "vector") sum of these parts is equal to E.

Example. Two coils b and c, Fig. 8, are connected in series between mains supplied from an alternator, of which the effective electromotive force is E. Then the total effective electromotive force E is subdivided into two parts E_1 and E_2, which act upon the respective coils, as indicated in Fig. 8; and the "geometric," or "vector" sum of E_1 and E_2 is equal to E, as shown in Fig. 9. The angles θ_1 and θ_2 depend upon the relative resistance and reactance of the respective coils. It is to be particularly noticed that the arithmetical sum of E_1 and E_2 is in general greater than E.

Physical Basis for the Differences between D.C. and A.C. Calculations. The above mentioned differences between direct-current and alternating-current calculations are due to the fact that an alternating current changes rapidly in value from instant to instant, while a direct current is more or less steady and does not change its direction of flow. A clear idea of the effects of the rapid changes of an alternating current may be obtained as follows:

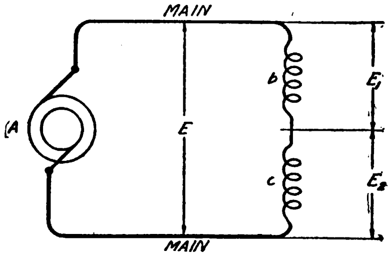


Fig. 8. Alternating Current Through Two Coils in Series

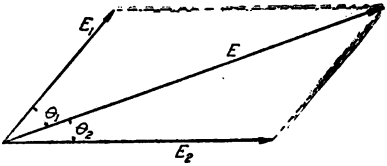


Fig. 9. Vector Diagram of E. M. F. for Conditions Shown in Fig. 8

Fig. 10 represents an alternator producing alternating current in a circuit of wire; and Fig. 11 represents a valveless pump, of which the piston oscillates rapidly up and down, producing an alternating current of water in a circuit of pipe. The current of water is called alternating because it is periodically changing its direction of flow according to whether the pump piston is pushing the column of water up or down. The e.m.f. of the alternator A not only has to overcome the resistance of the wire in order to cause an alternating current to surge back and forth through the circuit, but it also has to overcome the electrical inertia of the circuit—*first,* in getting a pulse of current started; and *second,* in stopping this pulse of current and starting another in the reverse direction. The pressure developed by the pump *P* not only has to overcome the frictional resistance of the pipe in order to cause an alternating current of water to surge back and forth through the pipe, but it also has to overcome the inertia of the water in the pipe—*first*, in getting a pulse of water current started; and *second*, in stopping this pulse and starting another in the reverse direction.

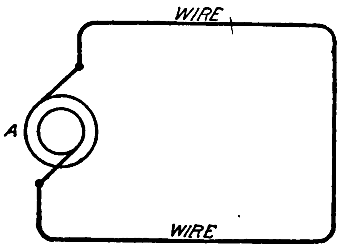


Fig. 10. Simple Alternating Circuit

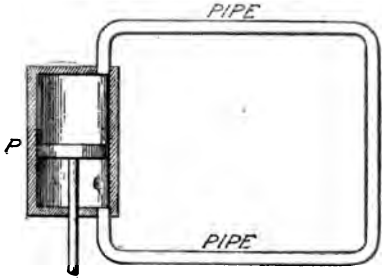


Fig. 11. Water Analogy for an Alternating Circuit

Fig. 12 represents an alternator producing alternating current in a circuit of wire which contains an electric condenser *C;* and Fig. 13 represents a valveless pump producing an alternating current of water in a circuit of pipe, which leads to a chamber *H H*, across which is stretched an elastic diaphragm *DD*. In this case the pressure developed by the pump has to overcome the frictional resistance of the pipe, the inertia of the water, and the elastic reaction of the diaphragm *DD*. Similarly the alternating electromotive force of the alternator *A*, Fig. 12, has to overcome the electrical resistance of the wire, the electrical inertia of the wire circuit, and the electro-elastic reaction of the dielectric, between the condenser plates.

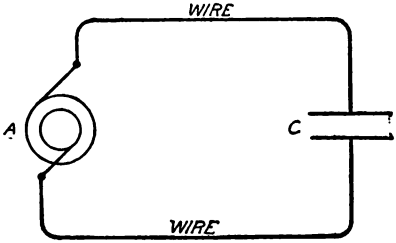


Fig. 12. Alternating Circuit Containing Condenser

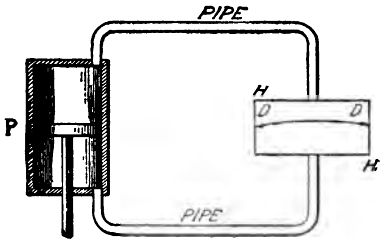
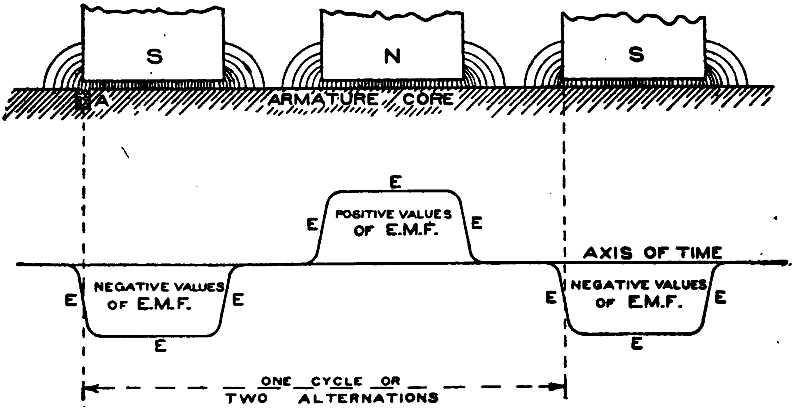


Fig. 13. Water Analogy for Alternating Circuit with Condenser

The electrical inertia of a circuit is called its *inductance* and the electro-elasticity of a condenser is called its *capacity,* and it is to inductance and capacity that the peculiar features of alternating-current calculations are due.

The effect of capacity is strikingly shown by the fact that an alternating current may be made to flow through a circuit which for direct currents would be an open circuit like that shown in Fig. 12. Thus an alternator connected to long transmission lines, which are disconnected at the distant end and perfectly insulated from the ground, will send a considerable alternating current into the lines, which current can be measured by an alternating-current ammeter. In such a case the current is called the *charging current* of the line; and it may amount to many amperes, according to the length of the line, the distance apart and size of wires, and the electromotive force of the alternator.


S
N
S
A
ARMATURE CORE
E
POSITIVE VALUES OF E.M.F.
E
E
AXIS OF TIME
NEGATIVE VALUES OF E.M.F.
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NEGATIVE VALUES OF E.M.F.
E
E
E
ONE CYCLE OR TWO ALTERNATIONS

Fig. 14. Development of Three Field Magnet Poles and E. M. F. Curve for One Cycle

Graphical Representation of Alternating Electromotive Forces and Currents. When an armature conductor of an alternator approaches a *north* pole of the field magnet, the electromotive force of the machine rises in value as the conductor enters the strong field under the pole; and the electromotive force falls in value as the conductor passes from under the pole. As the conductor passes the point midway between two adjacent poles, the electromotive force of the machine falls to zero, since no lines of the force are cut at this point. As the armature continues to revolve and the conductor approaches the next (the *south*) pole of the field magnet, the electromotive force of the machine again increases in value, but in a direction opposite to that of the previous electromotive force; and it falls again to zero as the conductor passes from under the south pole and reaches the point midway between the next pair of poles.

In Fig. 14 are represented the development of three successive poles *S*, *N*, *S* of the field magnet of an alternator, from which the lines of magnetic flux are emanating, and spreading out more or less as they enter the armature core. The armature core, also a developed view, is shown as having only one slot *A*, which contains a number of armature conductors. The ordinates of the curve *E E E E E E* represent the successive instantaneous values of the electromotive force induced in the armature conductors as the slot moves from left to right.

The duration of one cycle is indicated in the figure, and this cycle repeats itself as the conductors pass by successive *pairs* of field poles. Thus, in a ten-pole alternator, there would be five complete cycles, or five complete waves of the electromotive curve for each revolution. When a wave repeats itself after a definite time interval, it is called a *periodic wave.*

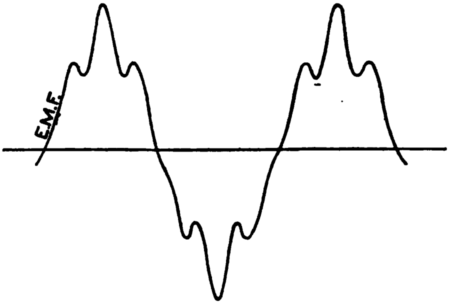

Fig. 15. Typical E. M. F. Curve for Alternator—"Peaked" Wave

The curve *E E E E E E* is called the *electromotive curve* or *electromotive force wave* of the alternator.

A curve of which the ordinates represent the successive instantaneous values of the alternating current and of which the abscissas represent time, is called an *alternating-current curve* or *alternating-current wave.*

Figs. 15, 16, and 17 show typical forms of electromotive force curves given by commercial alternators. Fig. 15 shows what is called a "peaked" wave; Fig. 16 shows a "flat-topped" wave; and Fig. 17 shows a "sine" or "sinusoidal" wave. All three waves are of course periodic. The exact shape of the electromotive force wave given by an alternator depends upon the relations between pole pitch (distance from center to center of adjacent poles), width, and shape of pole faces, width of armature coils, and distribution of coils on the armature. Alternators which give electromotive force waves approximating a sine wave, are preferred for power transmission.

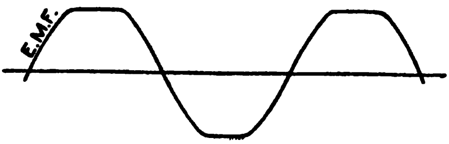

Fig. 16. Typical E. M. F. Curve for Alternator—"Flat Topped" Wave

Average and Effective Values of E. M. F. The average value of an alternating electromotive force or current *during a complete cycle* is zero, inasmuch as similar sets of positive and negative values occur.

The average value of an electromotive force or current *during the positive (or negative) part of a cycle* is usually spoken of briefly as the "average value" or "mean value," and is not zero.

Consider now an alternating current, of which the instantaneous value is i. The rate at which heat is generated in a circuit through which the current flows is i^2R, where R is the resistance of the circuit; and the *average* rate at which heat is generated in the circuit is R multiplied by the *average* value of i^2.

A continuous current which would produce the same heating effect would be one of which the square is equal to the average value of i^2, or of which the actual value is equal to $\sqrt{\text{average } i^2}$. This square root of the average square of an alternating current is called the *effective value* of the alternating current. Similarly, the square root of the average square of an alternating electromotive force is called the *effective value* of the alternating electromotive force. Voltmeters and ammeters used for measuring alternating electromotive force *or current* always give effective values irrespective of wave form; and in specifying an alternating electromotive force or current, its effective value is always used.

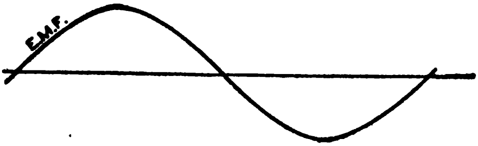

Fig. 17. Typical E. M. F. Curve for Alternator — "Sine" Wave

Example. Ten successive instantaneous values of an alternating electromotive force during half a cycle are 0, 30, 60, 80, 90, 95, 90, 80, 60, and 30 volts. The sum of these values is 615 volts, which, divided by the number of values, namely ten, gives 61.5 volts, which is the average value of this electromotive force during half a cycle.

Squaring each of the above values, adding the squares together, and dividing their sum by their number, namely ten, gives the average value of the square of the electromotive force, which is 4,702.5 volts^2; and the square root of this average square is 68.57 volts, which is the effective value of the given electromotive force.

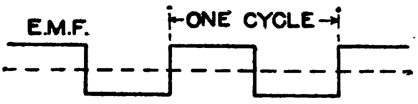

Fig. 18. Rectangular Form of E. M. F. Curve

Form Factor. The ratio *effective value ÷ average value*, depends upon the shape of the electromotive force wave, and is called the "form factor" of the wave.

Example. The form factor in the above case is $\frac{68.57}{61.5}$, or 1.115. The form factor of the electromotive force curve given in Fig. 17, which is a sine wave, is 1.11. The more peaked the wave the greater the value of its form factor. The rectangular electromotive force shown in Fig. 18 has a form factor equal to unity, which is the least possible value of the form factor. This rectangular wave, however, is never realized in commercial alternators.

Instantaneous and Average Power. Let e be the value, at a given instant, of the electromotive force of an alternator and let i be the value of the current at the same instant. Then ei is the power in watts which is delivered by the alternator at the given instant; and the average value of ei during a complete cycle is the average power delivered by the alternator.

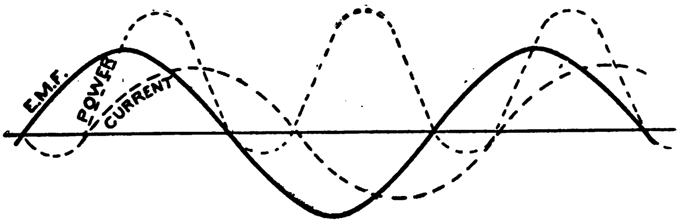

Fig. 19. E. M. F., Current, and Power Curves for an Alternator in a Circuit Containing Inductance

In Fig. 19 the full-line curve represents the electromotive force of an alternator and the heavy-dotted curve represents the current delivered by the alternator to a receiving circuit having inductance, such as an induction motor, for instance. The ordinates of the light-dotted curve represent the successive instantaneous values of the power *ei*. As shown in the figure, the power has both

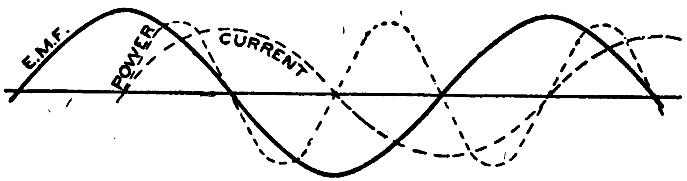

Fig. 20. E. M. F., Current, and Power Curves for Alternating Circuit with Large Inductance

positive and negative values; the alternator does work on the circuit when *ei* is positive, or is above the horizontal axis of time; and the circuit returns power to the alternator when *ei* is negative, or is below the horizontal axis of time; and this means of course, that while *ei* is negative, the dynamo is momentarily a motor and will be for the moment returning power to the fly wheel of the driving engine or turbine.

When the inductance of the receiving circuit is very large, the electromotive force and current curves are related as shown in Fig. 20; the instantaneous power *ei* passes through approximately similar sets of positive and negative values, as shown by the light-dotted curve; and the average power is approximately zero. This case would be very closely exemplified by an alternator connected to a transformer whose secondary was open-circuited, that is, supplying no current.

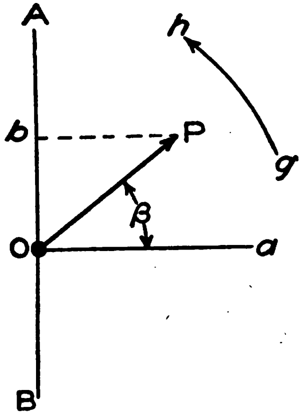

Fig. 21. Diagram of Harmonic E. M. F.s

Harmonic Electromotive Forces and Currents. A line *OP*, Fig. 21, revolves, at a uniform rate, *f* revolutions per second about a point *O*, in the direction of the arrow *gh*. Since the length of *OP* is fixed, the path or locus of the point *P* will be a circle about *O* as a center. Consider the projection *Ob* of this rotating line upon the fixed line *AB*, this projection being considered positive when above *O* and negative when below *O*.

A harmonic electromotive force (or current) is an electromotive force (or current) which is at each instant proportional to the line Ob.

The line Ob represents at each instant the actual value e of the harmonic electromotive force to a definite scale, and the length of the line OP, which is the maximum length of Ob, represents the maximum value E of the harmonic electromotive force to the same scale. The line Ob passes through a complete cycle of values during one revolution of OP, and so also does the harmonic electromotive force e. Therefore, the revolutions per second f of the line OP is the frequency of the harmonic electromotive force e. The rotating lines E and I, Fig. 22, of which the projections on a fixed line (not shown in the figure) represent the actual instantaneous values e and i of a harmonic electromotive force and a harmonic current, are said to "represent" the harmonic electromotive force and current, respectively. Of course, the rotation of the lines E and I is a thing merely to be imagined. The rotation is understood to be in a counter-clockwise direction, as indicated in Fig. 21.

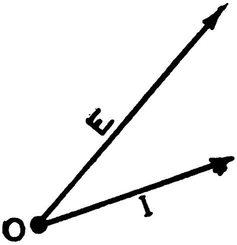

Fig. 22. Clock Diagram

Clock Diagram Representation. A diagram in which a number of electromotive forces or currents, or both, are represented by lines imagined to be revolving, is called a *clock diagram*. Simple problems involving relations between a number of *harmonic* electromotive forces and currents of the *same frequency*, are most easily treated by means of the clock diagram. The proper representation of alternating electromotive forces and currents in a clock diagram, requires that

(a) The given electromotive forces and currents be harmonic, and be of the same frequency.
(b) The lengths of the lines represent their maximum value to a suitable scale although the scales chosen for volts and for amperes may be different.
(c) The direction of the electromotive forces and currents be indicated by arrow heads.
(d) The relative position or phase of the electromotive forces and currents be constant and indicated by the angle between the various lines representing the given quantities.

When an electromotive force (or current) wave is not a sine curve, the electromotive force (or current) is not harmonic, and cannot properly be represented by a line in a clock diagram, because the projection of the rotating line is not at each instant proportional to the electromotive force (or current). The *approximate* representation, by lines in a clock diagram, of non-harmonic electromotive forces (or currents)—such, for example, as those represented by the curves in Figs. 15, 16, and 18, depends upon the finding of harmonic electromotive forces (or currents) which for the particular purpose in view are approximately equivalent to the actual given electromotive forces (or currents).

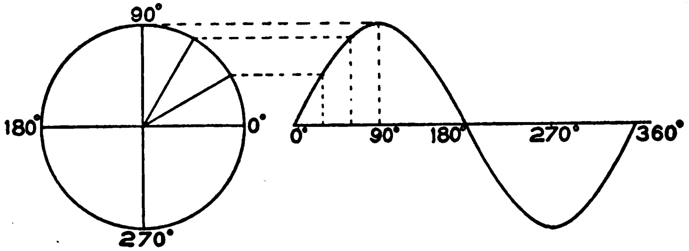

Fig. 23. Method of Plotting Sine E. M. F. Curve

Graphical Representation. A harmonic electromotive force or current is represented by a sine wave as shown in Fig. 17. The relation between the rotating line *OP* in Fig. 19 and the sine-wave curve of electromotive force is shown as follows:

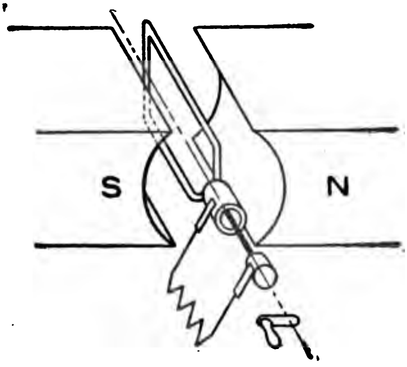

Fig. 24. Simple Dynamo Diagram

Divide the circumference of the circle in Fig. 23 into equal parts, and lay off a horizontal line divided into the same number of equal parts.

Draw horizontal dotted lines through each division on the circumference of the circle, and vertical dotted lines through the corresponding divisions on the horizontal line. The points of the intersection of these pairs of dotted lines are points on a curve which is a curve of sines.

A flat loop of wire with its terminals connected to two collecting rings gives a harmonic electromotive force when it is rotated at constant speed in a uniform magnetic field. This arrangement is shown in Fig. 24.

Algebraic Representation. The line OP, Fig. 21, revolves uniformly f revolutions per second and, therefore, it turns through $2\pi f$ radians* per second, since there are 2π radians in a revolution; that is

$$\omega = 2\pi f \tag{8}$$

in which ω is the angular velocity of the line OP in radians per second. Let time be reckoned from the instant that OP coincides with Oa; then, after t seconds, OP will have turned through the angle β $(=\omega t)$; and from Fig. 21 we have

$$Ob = OP \sin \beta = OP \sin \omega t$$

since Ob is the projection of OP on the line AB. But Ob represents the actual value e of the harmonic electromotive force at the time t, and OP represents its maximum value E; therefore

$$e = E \sin \omega t \tag{9}$$

is an algebraic expression for the actual value e of a harmonic electromotive force at time t, E being the maximum value of e, and $\frac{\omega}{2\pi}$ being the frequency according to equation (8).

Similarly

$$i = I \sin \omega t \tag{10}$$

is an algebraic expression for the actual value i of a harmonic current at time t, I being the maximum value of i.

If time is reckoned from the instant that OP, Fig. 21, coincides with the line Ob, then equations (9) and (10) become

$$e = E \cos \omega t$$
$$i = I \cos \omega t$$

Synchronism. Two alternating electromotive forces or currents are said to be in synchronism when they have the same frequency. Two alternators are said to run in synchronism when their electromotive forces and frequencies are similar.

Phase Difference. Consider two harmonic electromotive forces represented by the ordinates of the curves E_1 and E_2, Fig. 25. The electromotive force represented by the curve E_1 reaches its maximum value *before* the electromotive force represented by the curve E_2. The electromotive force E_1 is said to *lead*, or to be ahead of, the

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*The unit of angle chiefly used in mechanics and in all theoretical work is the *radian*. It is the angle of which the arc is numerically equal to the radius (of a circle). There are, therefore, 2π radians in one circumference.

electromotive force E_2 in *phase*. Conversely, the electromotive force E_2 is said to *lag behind* or to *follow* the electromotive force E_1 in phase. The same two electromotive forces E_1 and E_2 are also represented by the lines OE_1 and OE_2 in the clock diagram, Fig. 26. Here the line OE_2 is behind OE_1, since the imagined rotation about O as a center is counter-clockwise. The *phase difference* is the time interval θ in Fig. 25, or the angle θ between OE_1 and OE_2 in Fig. 26. If according to equation (9) the actual value of the harmonic electromotive force E_1 is $e_1 = E_1 \sin \omega t$, then the actual value of the electromotive force E_2, which lags θ degrees behind E_1, is $e_2 = E_2 \sin (\omega t - \theta)$. Similarly, if E_2 were taken as the reference line in the diagram, its actual value would be $e_2 = E_2 \sin \omega t$, and the value of E_1 would then be $e_1 = E_1 \sin (\omega t + \theta)$.

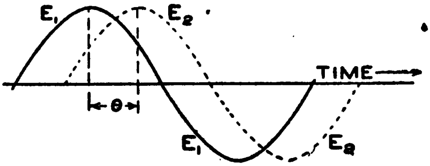

Fig. 25. Curves of Two Related E. M. F.s

When the angle θ, Fig. 26, is zero, as shown in Fig. 27, the electromotive forces E_1 and E_2 are said to be *in phase*. In this case the electromotive forces increase together and decrease together; that is, when E_1 is zero, E_2 is also zero; and when E_1 is at its maximum value, so also is E_2, etc. Therefore, $e_1 = E_1 \sin \omega t$ and $e_2 = E_2 \sin \omega t$.

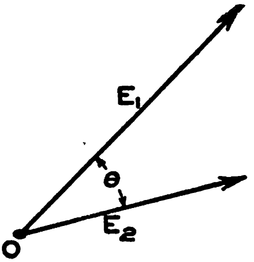

Fig. 26. Clock Diagram of E. M. F.s for Fig. 25

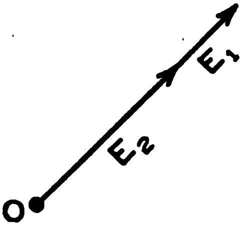

Fig. 27. Clock Diagram for Curves in Fig. 25 with E. M. F.s in Phase

When $\theta = 90°$, as shown in Fig. 28, the two electromotive forces are said to be *in quadrature*. In this case one electromotive force is zero when the other is a maximum, etc., or $e_1 = E_1 \sin \omega t$ and $e_2 = E_2 \sin (\omega t - \frac{\pi}{2})$.

When $\theta = 180°$, as shown in Fig. 29, the two electromotive forces are said to be *in opposition*. In this case they are at each instant opposite in sign; and when one is at its positive maximum, the other is at its negative maximum, etc. In this case $e_1 = E_1 \sin \omega t$ and $e_2 = E_2 \sin (\omega t \pm \pi)$. It is to be particularly noted that the principle of phase difference which has been illustrated in Figs. 24-27 for the case of two harmonic electromotive forces E_1 and E_2, applies equally to the case of two harmonic currents I_1 and I_2 and to the case of an electromotive force E_1 and a current I_1. Thus if in the clock diagram, Fig. 22, E represented a harmonic electromotive force having a maximum value of 1,000 volts, and I represented a harmonic current having a maximum value of 10 amperes with a phase difference of $\theta = 30°$ between them, the instantaneous values of E and I would be

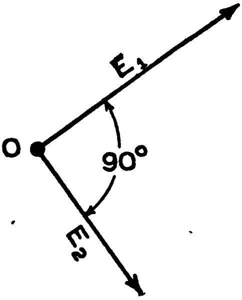

Fig. 28. Diagram of E. M. F.s of Fig. 25 in Quadrature

$$e = 1000 \sin \omega t$$

$$i = 10 \sin (\omega t - \frac{\pi}{6})$$

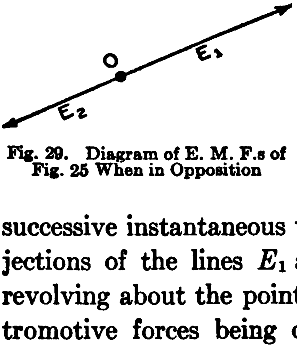

Fig. 29. Diagram of E. M. F.s of Fig. 25 When in Opposition

Addition of Harmonic Electromotive Forces and Currents. Consider two harmonic electromotive forces of which the successive instantaneous values e_1 and e_2 are represented by the projections of the lines E_1 and E_2, Fig. 30, which are imagined to be revolving about the point O. These electromotive forces being of the same frequency, the lines E_1 and E_2 revolve at the same speed, so that the angle between E_1 and E_2 remains unchanged in value. The ordinary arithmetical sum of e_1 and e_2, namely $e_1 + e_2$, is a harmonic electromotive force of the same frequency as e_1 and e_2; and this electromotive force $(e_1 + e_2)$ is represented by the projection of the line E, Fig. 30, which revolves at the same speed as E_1 and E_2.

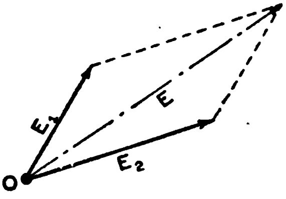

Fig. 30. Vector Diagram Showing Two Harmonic E. M. F.s

This is evident when we consider that the projection, on any line, of the diagonal of a parallelogram is equal to the sum of the projections of two adjacent sides of the parallelogram, as shown in Fig. 31. The projection of E_1 is Oc, which represents e_1; and the projection of E_2 is equal to cd, which represents e_2. The projection of the diagonal Ob of the parallelogram is Od, which is the sum of Oc and cd. The two lines marked E_2 in Fig. 31 are equal and parallel and have, therefore, the same projected length on the vertical line.

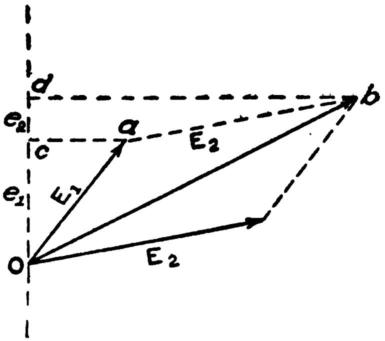

Fig. 31. Diagram Showing Addition of Harmonic E. M. F.s

As a *corollary* to the above, it may be stated that the ordinary arithmetical sum ($e_1 + e_2 + e_3 +$ etc.) of the instantaneous values of any number of harmonic electromotive forces (or currents) is another harmonic electromotive force (or current) of the same frequency; it is represented in magnitude and phase by a line that is the geometric (or vector) sum of the lines representing the given individual electromotive forces (or currents). This is evident when we consider that $e_1 + e_2$ is a harmonic electromotive force (or current) according to the above discussion; and this, added to e_3, gives an electromotive force (or current) which is harmonic and of the same frequency as e_1, e_2, and e_3.

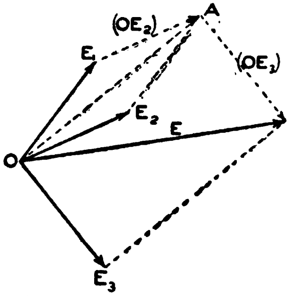

Fig. 32. Addition of E. M. F.s by Vector Polygon

The *geometric* or *vector sum* of a number of lines is obtained as follows: Given three lines OE_1, OE_2, and OE_3, Fig. 32. Find the diagonal OA of the parallelogram constructed on OE_1 and OE_2 as sides. This gives the vector sum of OE_1 and OE_2. Next construct a parallelogram on OA and OE_3 as sides; the diagonal OE of this parallelogram is the vector sum of the three given lines. This line OE is the closing side of the polygon formed by drawing OE_1, then drawing OE_2 from the extremity of OE_1 (this giving the point A), and then drawing OE_3 from A. This method is called *addition by means of the vector polygon.*

For example, two alternators A and B running in synchronism are connected *in series* between mains as shown in Fig. 33. If the electromotive forces of A and B are in phase, the electromotive force between the mains will be simply the numerical sum of the electromotive forces of A and B. If, on the other hand, the electromotive forces of A and B differ in phase, the state of affairs will be as represented in Fig. 34, in which the lines A and B represent the electromotive forces of the alternators A and B, respectively, θ is the phase difference of A and B, and the line E represents the electromotive force between the mains. The line E is the vector sum or resultant of A and B, and as shown is the diagonal of a parallelogram constructed on A and B as sides.

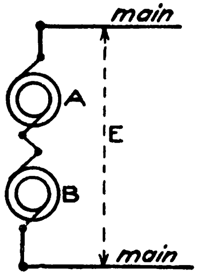

Fig. 33. Two Alternators Running in Synchronism

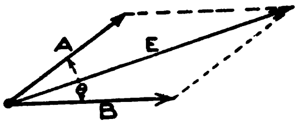

Fig. 34. Vector Diagram of Conditions in Fig. 33 if E. M. F.s Differ in Phase

Again, alternators A and B running in synchronism are connected *in parallel* between the mains as shown in Fig. 35. Let the lines A and B, Fig. 36, represent the currents given by the alternators A and B, respectively, the phase difference between the currents being θ; then the current in the main line is represented by I.

Consider the case of two circuits A and B, Fig. 37, connected *in series* between the mains of an alternator. The line E, Fig. 38, represents the electromotive force between the mains; the line A

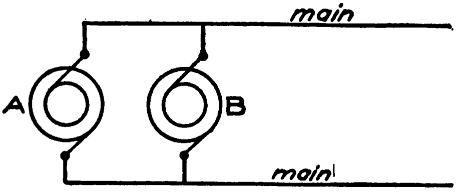

Fig. 35. Two Alternators Running in Synchronism Connected in Parallel

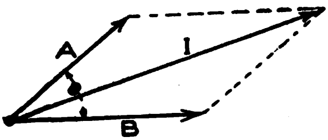

Fig. 36. Vector Diagram of Currents for the Condition of Fig. 35

represents the electromotive force between the terminals of the circuit A; and the line B represents the electromotive force between the terminals of the circuit B. The circuits A and B are supposed to have inductance. If either of the circuits contains a condenser, then the electromotive forces A and B, Fig. 38, may be nearly opposite to each other in phase, and A and B may each be indefinitely greater than the electromotive force E between the mains.

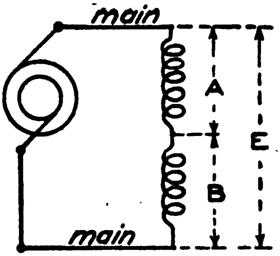

Fig. 37. Diagram of Two Coils in Series with an Alternator

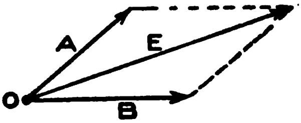

Fig. 38. Diagram of E. M. F.s for Conditions Shown in Fig. 37

Again, two circuits A and B, Fig. 39, are connected *in parallel* across the terminals of an alternator as shown. The current I from the alternator is related to the currents A and B as shown in Fig. 40. If either of the circuits A or B contains a condenser, then the currents A and B may be nearly opposite to each other in phase, and the currents A and B may each be indefinitely greater than the current I from the alternator.

Subtraction of Harmonic Electromotive Forces and Currents. One harmonic electromotive force (or current) is subtracted from another by reversing the direction of the line that represents it in the clock diagram, and then *adding* the reversed line (or vector) to the other. An example of the subtraction of harmonic electromotive forces will be given in connection with the discussion of three-phase electromotive force.

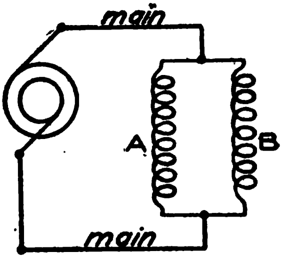

Fig. 39. Coils in Parallel with an Alternator

Relation between Maximum and Effective Values. The effective value E or I of a harmonic, electromotive force, or current—that is, one whose graph is a sine wave—is equal to the maximum value $\mathbf{E}$ or $\mathbf{I}$ divided by the square root of *2*. That is

$$E = \frac{\mathbf{E}}{\sqrt{2}} \tag{11}$$

$$I = \frac{\mathbf{I}}{\sqrt{2}} \tag{12}$$

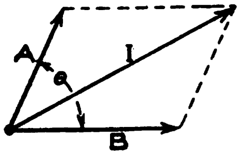

Fig. 40. Current Diagram of the Conditions Shown in Fig. 39

This may be shown as follows: Let e $(= \mathbf{E} \sin \omega t)$ be a harmonic electromotive force. To find the average value of e^2 $(= \mathbf{E}^2 \sin^2 \omega t)$, it is necessary to find the average value of the square of the sine of the uniformly variable angle ωt. We have the general relation

$$\text{(a)} \qquad \sin^2 \omega t + \cos^2 \omega t = 1$$

so that

$$\text{(b)} \quad \text{Av.} \sin^2 \omega t + \text{Av.} \cos^2 \omega t = 1$$

Now, during a cycle, the cosine of a uniformly variable angle passes similarly through the same set of values as the sine; hence Av. $\sin^2 \omega t$ and Av. $\cos^2 \omega t$ are equal, so that from equation (b) above, we have

$$2 \text{ Av.} \sin^2 \omega t = 1$$

or

$$\text{Av.} \sin^2 \omega t = \tfrac{1}{2}$$

The average value of e^2 is

$$\text{Av. } e^2 = \mathbf{E}^2 \text{ Av.} \sin^2 \omega t$$

or

$$\text{Av. } e^2 = \frac{\mathbf{E}^2}{2}$$

and

$$\sqrt{\text{Av. } e^2} = \frac{\mathbf{E}}{\sqrt{2}}$$

In Fig. 21 the length of the revolving line OP was understood to represent the *maximum value* of the harmonic electromotive force (or current). When, however, a number of harmonic electromotive forces (or currents) are represented to scale by lines in a clock diagram, the lengths of the lines may be interpreted as giving not maximum but *effective values*, since there is a constant ratio $\sqrt{2}$ between the effective and the maximum values of each of the electromotive forces (or currents) represented in the diagram.

NOTE. It is desirable to interpret the lines in a clock diagram in terms of effective values rather than maximum values, because effective values are always given by measuring instruments and are nearly always used in numerical calculation. Therefore, unless it is expressly stated to the contrary, the lines in clock diagrams are always understood to represent effective values.

For example, a certain harmonic alternating current gives a reading of 100 amperes on an alternating-current ammeter, and its effective value is, therefore, 100 amperes. This harmonic current actually pulsates between zero and a maximum value of $\pm\sqrt{2} \times 100$ amperes, or $\pm$ 141.4 amperes.

Again, a certain harmonic alternating electromotive force gives a reading of 1,000 volts on an alternating-current voltmeter, and its effective value is, therefore 1,000 volts. This electromotive force actually varies between zero and a maximum value of $\pm\sqrt{2} \times$ 1,000 volts, or $\pm$1,414 volts.

The above simple relation between maximum and effective values is true only for harmonic (that is, sine-wave) electromotive forces and currents. In general, the maximum values of alternating electromotive forces or currents cannot be inferred from effective values as measured by voltmeters or ammeters. Thus, an alternating electromotive force which is known to have a peaked-wave form might have a maximum value very greatly in excess of $\sqrt{2}$ times its effective value.

Expression for Power. (a) When the current is in phase with the electromotive force, that is, when the circuit is non-inductive, then the power (average *ei*, see page 22) is

$$P = EI \tag{13}$$

in which P is the power in watts, E is the effective value of the electromotive force in volts, and I is the effective value of the current in amperes. Equation (13) is identical with the power equation for direct-current circuits.

(b) If the phase difference between current and electromotive force in a given circuit were 90 degrees, which can never actually occur, then the power (average value of *ei*) would be equal to zero, as explained on page 23.

(c) When the phase difference between current and electromotive force is $\theta°$, as shown in Fig. 41, then

$$P = EI \cos\theta \qquad (14)$$

in which P is the power (average ei) in watts, E is the effective value of the electromotive force in volts, and I is the effective value of the current in amperes.

For example, the given current I shown in Fig. 41 may be thought of as resolved into two components, as shown in Fig. 42. One of these components $I \cos\theta$ is parallel to (that is, in phase with) E; and the other $I \sin\theta$ is at right angles to E. The power corresponding to the actual current I may be thought of as the sum of the powers corresponding to its two components, respectively. But the component $I \sin\theta$ is at right angles to E, as in (b) above; hence the power corresponding to it is zero. This component is, therefore, often called the ***wattless component*** or, more correctly, the ***reactive component*** of the given current.

On the other hand, the component $I \cos\theta$ is parallel to (that is, in phase with) E, as in (a) above; hence the power corresponding to this component is equal to $E \times I \cos\theta$. The component $I \cos\theta$ of the given current I is called the ***power component*** or, more correctly, the ***active component*** of I, and the factor $\cos\theta$ the ***power factor*** of the circuit.

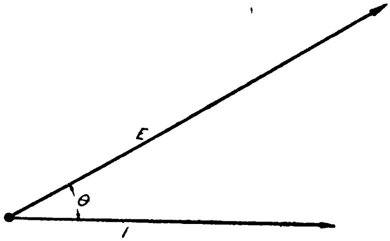

Fig. 41. Diagram Showing Phase Difference Between E. M. F. and Current

Inductance. It has been pointed out, page 18, that an electric circuit has a certain kind of inertia analogous to the inertia of water in a circuit of pipe, and it was there noted that this inertia of an electric circuit is called inductance. If an electric current in a circuit is made to change in value, a portion of the electromotive force acting upon the circuit must be used to *cause* the current to change.

In the same way a force over and above that required to overcome frictional resistance must act upon a moving body to accelerate it, that is, to make its speed increase. The inertia of a body is measured by the force required to accelerate it at the rate of unit change in speed per second; and the inductance of a circuit is measured by the electromotive force required to cause a current in the circuit to change at the rate of one ampere per second. A circuit is said to have an inductance of one *henry** when one volt (over and above the electromotive force required to overcome the electrical resistance) will cause the current to change at the rate of one ampere per second.

Let x be the rate in amperes per second at which the current in a circuit is increasing in value. Then the electromotive force E (over and above that required to overcome the resistance of the circuit) required to cause the current to increase at this rate is

$$E = Lx \tag{15}$$

in which L is the inductance of the circuit in henrys, E being expressed in volts.

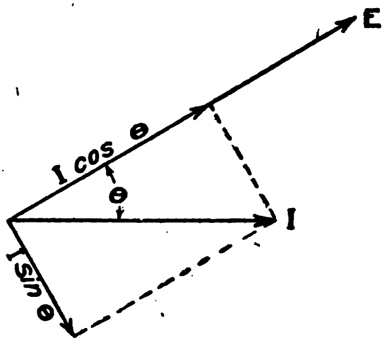

Fig. 42. Resolution of the I of Fig. 41 into Two Components

Example. The coil of a certain large electromagnet has 2.5 henrys of inductance and 5 ohms of resistance. At a given instant this coil is connected to 110-volt direct-current mains. At the instant of connecting the coil, the current is zero, and all of the 110 volts is used to cause the current in the coil to increase, so that, according to equation (15), x is equal to $\frac{E}{L}$, or 110 volts ÷ 2.5 henrys, or 44 amperes per second. That is, the current in the magnet coil begins to increase at the rate of 44 amperes per second. When the current in the magnet coil has reached the value of 10 amperes, 50 volts (= 5 ohms × 10 amperes) of the total 110 volts are used in overcoming the resistance of the coil, so that 60 volts (= 110 volts − 50 volts) are used to make the current increase. Therefore, as the current in the coil passes the value of 10 amperes, it is increasing at the rate $\frac{E}{L}$, or 60 volts ÷ 2.5 henrys, or 24 amperes per second.

The inductance of a coil wound on a given spool is proportional to the square of the number of turns N of wire. For example, a given spool wound with No. 16 wire has 500 turns and an inductance of, say, 0.0025 henry; the same spool wound with No. 28 wire would have about eight times as many turns, and its inductance would be about 64 times as great, or 0.16 henry.

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*The henry is a very large inductance, and the inductances usually met with in practice are expressed in thousandths of a henry, that is, in milli-henrys.

The inductance of a coil of given shape is proportional to its linear dimensions, the number of turns of wire being unchanged. For example, a given coil has an inductance of 0.022 henry; and a coil three times as large in length, diameter, etc., but having the same number of turns of wire, has an inductance of 0.066 henry.

To Prove $L=\frac{\Phi N}{I}$. In Fig. 43 is shown a coil of N turns containing negligible resistance and carrying a direct current of I amperes which sets up a flux of Φ lines of force. If the number of turns remains fixed and there is no iron in the circuit, the value of Φ varies directly with the current I. Thus for every change of one ampere the value of Φ will change in a corresponding manner by an amount $\frac{\Phi}{I}$ (flux per ampere) which is a constant depending on the size, shape, number, and arrangement of the coils if no iron is present. If for any reason the current flowing from a to b should increase at the rate of 1 ampere per second, it is obvious that the flux through the coil will increase at the rate of $\frac{\Phi}{I}$ lines per second and in each turn of the coil there will be induced $\frac{\Phi}{I}$ ab volts, or $\frac{\Phi}{10^8 I}$ volts, in such a direction as to oppose the increase in current; or, according to Lenz's law, from b to a. Since the foregoing value of voltage is induced in each of the N turns and the turns are in series, the total induced voltage from b to a will be $\frac{N\Phi}{10^8 I}$ volts. If for any reason the current I from a to b should decrease at the rate of 1 ampere per second, a voltage of $\frac{N\Phi}{10^8 I}$ volts would be induced in a direction to oppose the decrease, or from a to b.

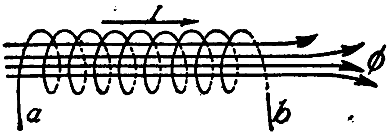

Fig. 43. Diagram Showing Inductance in Coil without Iron Core

Should the current change at any other rate than 1 ampere per second, the induced voltage would be numerically equal to $\frac{N\Phi}{10^8 I}$ times the rate of change of current in amperes per second. Let us choose the positive direction of current from *a* to *b*. Then for a positive change, or increase, in current the induced voltage is negative and for a negative change, or decrease, in current the induced voltage is positive, so that to properly represent the induced voltage in magnitude and direction it is necessary to write

$$e \text{ (induced)} = \frac{N\Phi}{10^8 I} \text{ times the rate of change of } I$$

It so happens that $\frac{N\Phi}{10^8 I}$ is a constant for any one particular coil and is represented by the symbol L commonly known as the inductance. Thus

$$e = -L \text{ times the rate of change of } I$$

From what has been said in preceding paragraphs it might be inferred that inductance has a tendency to keep the current at a constant value, and this is correct; for, should the terminals *a* and *b* be joined by a conductor of zero resistance, the current I would flow forever. To make I increase in the direction of *a* to *b* would require a voltage in this direction, and the rate of increase of I caused by this applied voltage would be such a value as to make the induced voltage exactly equal and opposite to the applied voltage. This same balancing effect should and will exist if a voltage is applied from *b* to *a* to cause the current to decrease. Thus we can say

$$e \text{ (applied)} = L \text{ times the rate of change of } I$$

Thus a positive applied voltage causes a positive rate of change of I, and a negative applied voltage causes a negative rate of change of I.

Example. A current of 5 amperes is passed through a coil of 1000 turns and produces 100,000 lines of flux. Calculate the inductance in henrys, assuming that no iron is present.

SOLUTION. $$L = \frac{\Phi N}{10^8 I} = \frac{100000 \times 1000}{10^8 \times 5} = 0.2 \text{ henry}$$

Formulas for Inductance. The inductance in henrys of a coil of wire wound in a thin layer on a long wooden cylinder having a length of l centimeters and a radius of r centimeters may be proved to be

$$L=\frac{4\pi^2 r^2 N^2}{l\times 10^9} \tag{16}$$

in which N is the total number of turns of wire in the coil. This equation is strictly true for very long coils wound in a thin layer; but the equation is also very useful in calculating the approximate inductance of even short, thick coils. Thus, a coil having a length of 25 centimeters and a mean radius of 2½ centimeters, containing 150 turns of wire, has from equation (16) an approximate inductance of

$$L=\frac{4\pi^2 (2.5)^2\times 150^2}{25\times 10^9}=0.00022 \text{ henry}$$

If a coil of N turns of wire is wound on a long rod of iron, instead of wood or other non-magnetic material, the inductance in henrys is given by the equation

$$L=\frac{4\pi^2 r^2 N^2 \mu}{l\times 10^9} \tag{17}$$

in which L, r, N, and l are the same as in equation (16), and μ is the permeability of the iron core at the particular flux density produced in the iron. This equation applies to any iron rod of length l centimeters and radius r centimeters, wound with N turns of wire, whether in the form of a long, straight rod or bent into a closed ring.

The permeability μ of iron varies from 500 to 1,000 or more; and, therefore, the effect* of placing an iron core in a coil is to increase greatly the inductance of the coil.

The iron core of an inductance coil to be used with alternating currents should be laminated to reduce eddy currents and the consequent loss of energy, and to prevent excessive heating of the core.

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* The permeability μ of a given sample of iron is not constant, but decreases in value as the magnetizing force increases. Therefore, the inductance L of a coil having an iron core is not a definite constant quantity as is the inductance of a coil without an iron core but depends upon the magnetizing current.

For example, the inductance of the field coil of a certain shunt-wound dynamo is 7.5 henrys. The inductance of a pair of No. 0, B. & S. copper line wires carried at a distance of 18 inches apart on a pole line is 0.0035 henry per mile. The inductance of the secondary coil of a large induction coil (X-ray coil) having 200,000 turns of wire, is 2,000 henrys.

Series and Parallel. The inductance of two or more coils in series is equal to the sum of the individual inductances.

The equivalent inductance of two or more similar coils in parallel, such as the similar coils on an armature, is equal to $\frac{1}{n}$ of the inductance of one coil, n being the number of coils connected in parallel.

Capacity. It has been pointed out, page 18, that a charged condenser has an elastic-like reaction analogous to the elastic reaction of a distorted diaphragm. The elasticity of a diaphragm might be measured by the pressure required to distort it to the extent of producing one unit of increase of volume in the space on one side of the diaphragm and one unit of decrease of volume in the space on the other side of the diaphragm. Similarly, the capacity of a condenser may be, and in fact is, measured by the electromotive force required to force one unit of charge of electricity into one plate of the condenser, and at the same time to withdraw one unit of charge from the other plate. A condenser is said to have a capacity of 1 *farad** when one volt of electromotive force pushes one coulomb of electric charge into the condenser.

The charge Q pushed into a condenser by a steady electromotive force E is

$$Q=CE \tag{18}$$

in which C is the capacity of the condenser in farads, Q is the charge in coulombs, and E is the electromotive force in volts. The electromotive force required to hold a given charge Q in the condenser is of course equal to $\frac{Q}{C}$.

Condensers, to have a large capacity (as much as a microfarad), are usually made up of alternate sheets of tinfoil and waxed paper,

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*The farad is an exceedingly large capacity, and capacities encountered in practice are usually expressed in millionths of a farad, that is, in microfarads.

TABLE II

Inductivities of Dielectrics

Referred to Air as Unity

Glass	3.00 to 10.00	Mica	4.00 to 8.00
Vulcanite	2.50	Shellac	2.93 to 3.3
Paraffin	1.68 to 2.30	Turpentine	2.15 to 2.43
Beeswax	1.86	Petroleum	2.02 to 2.19
Gutta-percha	3.3 to 4.9	Rubber (pure India)	2.12 to 2.24

or mica, as indicated in Fig. 44. Alternate metal sheets are connected together giving two terminals as shown, thus practically forming two metallic plates of large area. A condenser having a capacity of 1 microfarad contains about 3,600 square inches of tinfoil.

Inductivity of Dielectric. The capacity of a condenser of given dimensions depends upon the material used between the plates, called the *dielectric*. The quotient, capacity of a condenser with given dielectric ÷ its capacity with air as the dielectric, is called the *inductivity* or the *specific inductive capacity* of the given dielectric. The values of the inductivity for the most commonly used dielectrics are given in Table II.

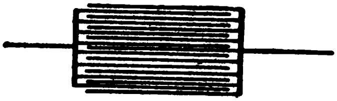


Fig. 44. Diagram of Condenser

Capacity of Condensers. The capacity of a condenser is given by the equation

$$C = \frac{885\,kA}{10^{10}\,d} \qquad (19)$$

in which C is the capacity in microfarads, k is the inductivity of the dielectric used, d is the distance in centimeters between plates, *i. e.*, the thickness of the dielectric, and A is the area in square centimeters of both sides of all the inner plates plus the area of the inner surfaces of the two outside plates.

NOTE. If the total number of plates $= n$, A will be the area of both sides of $(n-1)$ plates; or, in other words, it is necessary to provide one more plate than is necessary, using both sides, to equal an area of A square centimeters.

Example. It is required to design a plate condenser to have a capacity of 1.5 microfarads using a dielectric of oiled paper 0.0043 inch thick and tinfoil 0.0007 inch thick.

Assuming for the oiled paper an inductivity of 2.67, equation (19) gives for the total active plate area

$$A=\frac{1.5\times10^{10}\times0.0043\times2.54}{885\times2.67}=69400 \text{ square centimeters}=10759 \text{ square inches}$$

If 65 plates be used, the area of one side of each plate will be

$$\frac{10759}{2(65-1)}=84 \text{ square inches}$$

so that the dimensions of one active plate may be $10\frac{1}{2}'' \times 8''$ and the area of tinfoil needed will be $84\times65=5460$ square inches.

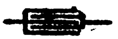
The capacity of an ordinary 2-quart Leyden jar is about 0.005 microfarad. The capacity of an average submarine telegraph cable is about 0.4 microfarad per nautical mile. The capacity of a pair of transmission lines of No. 0, B. & S. wires placed 18 inches apart between centers on poles, is 0.036 microfarad per mile.

Series and Parallel. The capacity of a number of condensers *in parallel* is equal to the sum of the individual capacities.

The capacity of a number of condensers *in series* is

$$C=\frac{1}{\frac{1}{C_1}+\frac{1}{C_2}+\frac{1}{C_3}+\text{ etc.}} \qquad (20)$$

in which C_1, C_2, C_3, etc., are the individual capacities, and C is the joint capacity.

Fundamental Equations of the A. C. Circuit. An alternator A, Fig. 45, delivers an alternating current of I amperes, effective, to a circuit consisting of a resistance of R ohms, an inductance of L henrys, and a condenser of which the capacity is C farads, all connected in series. It is to be remembered that any coil having inductance has resistance also; that is, inductance and resistance are practically inseparable. Nevertheless inductance and resistance are essentially different in nature and in their effects, and they are always considered separately, so that it is helpful to think of them as actually separated in a circuit, as indicated in Fig. 45. A resistance is conventionally represented thus, ; an inductance thus, ; and a condenser thus, .

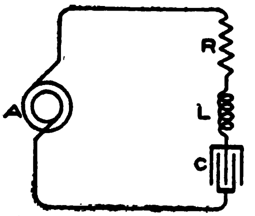


Fig. 45. Alternating Circuit Containing Resistance, Inductance, and Capacity

The current in the circuit, Fig. 45, is assumed to be harmonic, that is, to be a sine-wave current, and this current is represented by the line OI in Fig. 46.

A portion of the electromotive force of the alternator is used to overcome the resistance of the circuit. The portion of the electromotive force so used is an alternating electromotive force of which the effective value is RI; it is in phase with the current, and is represented by the line RI in Fig. 46.

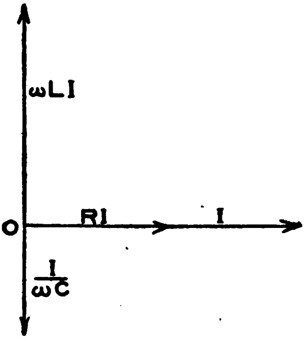


Fig. 46. Vector Diagram of Conditions in Fig. 45

A portion of the electromotive force of the alternator is used to overcome the inertia or inductance of the circuit in causing the current to increase and decrease. The portion of the electromotive force so used is an alternating electromotive force of which the effective value is ωLI; it is 90 degrees ahead of I in phase, and is represented by the line ωLI in Fig. 46. The quantity ω is equal to 2π times the frequency of the current I.

A portion of the electromotive force of the alternator is used to overcome what we have previously called the electro-elasticity of the condenser, or, in other words, to hold electric charge on the condenser plates at each instant. The portion of the electromotive force so used is an alternating electromotive force of which the effective value is $\frac{I}{\omega C}$; it is 90 degrees behind I in phase, and is represented by the line $\frac{I}{\omega C}$ in Fig. 46.

The total electromotive force E of the alternator is equal to the geometric (or vector) sum of the parts RI, ωLI, and $\frac{I}{\omega C}$. This vector sum is formed by subtracting $\frac{I}{\omega C}$ from ωLI, since it is opposite to ωLI in direction, and then adding RI and $(\omega LI - \frac{I}{\omega C})$ geometrically, as shown in Fig. 47, in which the line Oa represents $(\omega LI - \frac{I}{\omega C})$

and the line E represents the geometric sum of Oa and RI.

From Fig. 47 we have, by geometry:

$$E^2 = R^2 I^2 + \left(\omega LI - \frac{I}{\omega C}\right)^2$$

or

$$E^2 = I^2 \left[R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2\right]$$

or

$$I = \frac{E}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}} \tag{21}$$

The quantity $\omega L - \frac{1}{\omega C}$ is called the *reactance* of the circuit. The term ωL is often called *inductance reactance;* and the term $\frac{1}{\omega C}$ is often called *capacity reactance.* Inductance reactance is always positive, and capacity reactance is always negative. It is convenient to represent the reactance $\omega L - \frac{1}{\omega C}$ of a circuit by the single letter X; that is

$$X = \omega L - \frac{1}{\omega C} \tag{22}$$

Therefore, writing X for $\omega L - \frac{1}{\omega C}$ in equation 21, we have

$$I = \frac{E}{\sqrt{R^2 + X^2}} \tag{23}$$

Furthermore, from the right triangle in Fig. 47 we have

$$\tan \theta = \frac{\omega L - \frac{1}{\omega C}}{R}$$

or

$$\tan \theta = \frac{X}{R} \tag{24}$$

in which θ is the angle of phase lag of the current I behind the electromotive force E; X is the reactance of the circuit; and R is the resistance of the circuit.

Resistance, Reactance, and Impedance. Consider a harmonic alternating electromotive force E which produces a harmonic alternating current I in a circuit. This electromotive force may be resolved into two components, one parallel and the other perpendicular to I, as shown, for example, in Fig. 46. The component of E parallel to I is equal to RI.

The resistance of an alternating-current circuit is sometimes defined as *that factor which, multiplied by the current, gives the component (of the electromotive force) which is parallel to I.*

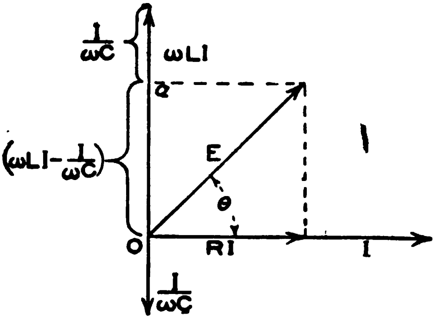


Fig. 47. Complete Diagram of Conditions in Fig. 45

The component of E perpendicular to I is equal to $\omega LI-\frac{I}{\omega C}$, or is equal to XI.

The reactance of an alternating-current circuit may be defined as *that factor which, multiplied by the current, gives the component (of the electromotive force) which is perpendicular to I.*

The factor $\sqrt{R^2+X^2}$, which, when multiplied by the current I, gives the total value of the electromotive force E, is called the impedance (denoted by Z) of the alternating-current circuit. Of course, E divided by Z gives the value of the current I.

Note. Resistance, reactance, and impedance are all expressed in ohms; we may, for example, speak of 10 ohms of resistance, 10 ohms of reactance, or 10 ohms of impedance. Thus, ohms are used in alternating-current work to express the three essentially different things—resistance, reactance, and impedance; and a specification of a certain number of ohms is not intelligible unless it is stated whether it is ohms of resistance, ohms of reactance, or ohms of impedance.

The reactance and the impedance of a circuit depend upon the frequency of the alternating current, as well as upon the physical constants L and C of the circuit, since the factor ω is equal to 2π times the frequency.

The reactance of a circuit may be positive or negative, according as ωL is larger than or less than $\frac{1}{\omega C}$. When reactance is positive, the inductance reactance ωL exceeds the capacity reactance $\frac{1}{\omega C}$, and the current is behind the electromotive force in phase, as shown in Fig. 47. When the total reactance is negative, however, the capacity reactance exceeds the inductance reactance, and the current is ahead of the electromotive force in phase, as shown in Fig. 48. The same results may be obtained from equation (24). If the total reactance X is negative, then tan θ is negative, which means that θ is a negative angle, or that the electromotive force is behind the current in phase, or that the current is ahead of the electromotive force.

Special Cases of Electromotive Force and Current Relations. A clear understanding of the following examples as special cases of the general relations of electromotive force and current as discussed on pages 42 and 43 depends upon the following facts:

(a) That the effect of inductance in an alternating-current circuit becomes negligible when the inductance is very small, for then the reactance ωL due to the inductance is small, and the portion of the electromotive force required to overcome the inductance, namely, ωLI, Fig. 46, is also small.

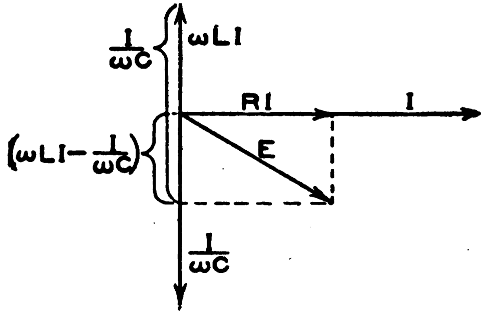


Fig. 48. Conditions of Fig. 45 When Total Reactance is Negative

(b) That the effect of a condenser in an alternating-current circuit becomes negligible only when the capacity of the condenser is very large, for then the reactance $-\frac{I}{\omega C}$ due to the condenser is small, and the portion of the electromotive force required to overcome the electro-elasticity of the condenser, namely, $\frac{I}{\omega C}$, Fig 46, is also small.

The effect of an inductance may be rendered negligible by short-circuiting it with a low-resistance wire; and the effect of a condenser also may be rendered negligible by short-circuiting it with a low-resistance wire.

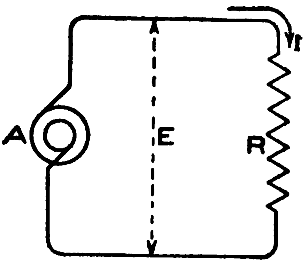


Fig. 49. Diagram of a Non-Reactive Alternating Circuit

Case A. *Non-inductive or non-reactive circuits.* A circuit which does not contain a condenser and does not have any perceptible inductance is called a *non-reactive circuit.* The term *non-inductive* is frequently used in the sense in which non-reactive is here defined. A non-reactive circuit contains only resistance; and the total electromotive force required to produce a given alternating current I in a non-reactive circuit of which the resistance is R ohms, is RI volts,* and the electromotive force and current are in phase with each other. Therefore, the relation between alternating electromotive force and current in a non-reactive circuit is precisely the same as in the case of direct currents. That is

$$E = RI$$

or

$$I = \frac{E}{R} \qquad (25)$$

Fig. 49 represents a non-reactive circuit connected to an alternator A; and Fig. 50 shows the relation between the electromotive force and current.

Any circuit in which the outgoing and returning wires are very near together, has very small inductance. An ordinary incandescent lamp, for example, has a negligible inductance. An incandescent lamp the resistance of which when hot is 220 ohms, takes half an ampere, effective, when connected to alternating-current supply mains between which the effective electromotive force is 110 volts; the current is in phase with the electromotive force and the power in watts is equal to the product of effective volts times effective amperes, or 55 watts. Alternating-current voltmeters are always made as nearly as possible non-inductive.

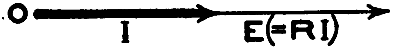


Fig. 50. E. M. F. and Current Relations for a Non-Reactive Alternating Circuit

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*Effective values are always understood except where it is distinctly stated to the contrary.

Case B. *Circuits containing resistance and inductance.* In this case the reactance X ($=\omega L$) is positive, and the current lags behind the electromotive force in phase, as before pointed out. The tangent of the angle of lag is equal to $\frac{X}{R}$, according to equation (24); therefore, the angle of lag of the current is small when X is small compared with R, and the angle of lag approaches 90° when X is very large compared with R.

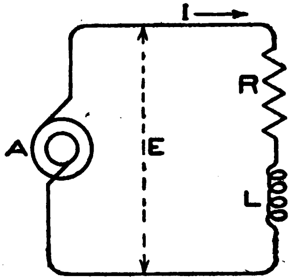

Fig. 51. Alternating Circuit Containing Resistance and Inductance

Fig. 51 represents a circuit containing resistance and inductance, connected to an alternator A; and Fig. 52 shows the relation between the electromotive force and current.

Examples. 1. A non-inductive resistance takes 10 amperes from 220 volt, 60-cycle mains. What current will it take: (a) from 220-volt, 25-cycle mains; (b) from 110-volt, 60-cycle mains?

Since the resistance is non-inductive, the impedance is equal to the resistance, and $I=\frac{E}{R}$. Solving for R we obtain $R=\frac{220}{10}=22$ ohms.

(a) The current will be $I=\frac{220}{22}=10$ amperes.

(b) Since there is no inductance or capacity in the circuit, the impedance is independent of the frequency. Therefore, $I=\frac{110}{22}=5$ amperes.

2. An impedance coil of negligible resistance takes 3 amperes from 220-volt, 60-cycle mains. What current will it take (a) from 220-volt, 25-cycle mains? (b) from 110-volt, 60-cycle mains?

In this case the resistance R is zero, so that the impedance is equal to the reactance, and $I=\frac{E}{X}$. Solving for X we obtain $X=\frac{220}{3}=73.3$ ohms when the frequency is 60 cycles.

(a) When the frequency is reduced from 60 to 25 cycles, the reactance X is reduced in the same ratio since $X=2\pi fL$. Therefore, the reactance at 25 cycles is $73.3\times\frac{25}{60}$ ohms. The current is $I=\frac{220\times 60}{73.3\times 25}=7.2$ amperes.

(b) Since frequency is again 60 cycles, $X=73.3$, and $I=\frac{110}{73.3}=1.5$ amperes.

A coil of wire usually has a very considerable inductance, especially if it is wound on a laminated iron core. In fact, a coil wound on a laminated iron core usually has so large a reactance X $(=\omega L)$, that the angle θ, Fig. 52, is very nearly 90°.

Example. A certain coil has a resistance of 2 ohms and an inductance of 0.3 henry when provided with a laminated-iron core. This coil is connected to an alternator giving 1,000 volts effective electromotive force at a frequency of 133 cycles per second, so that the factor ω is equal to $2\pi \times 133$, or 835.7 radians per second; the reactance of the coil is 835.7×0.3, or 250.7 ohms; the impedance is $\sqrt{2^2 + 250.7^2}$ or 250.7 ohms; the current is $\frac{1{,}000}{250.7}$, or 3.989 amperes; the current lags about 89½° behind the electromotive force; and the power delivered to the coil is 1,000 volts × 3.989 amperes × cos 89½° (0.008), which is equal to 31.9 watts $(=I^2R)$. The product EI, sometimes called *apparent watts*, is equal to 3,989 volt amperes.

This example illustrates one remarkable feature of alternating currents—namely, the very small amount of actual power that is delivered to a circuit of large reactance even though the electromotive force is large and the current considerable. In the case of a direct current, 3.989 amperes taken from 1,000-volt mains would mean an actual delivery of 3,989 watts of power, while in the above case the actual power delivered is only 31.9 watts. The ratio *true watts* ÷ *apparent watts* is called the *power factor* of a circuit; and in case of the coil here under discussion, this ratio is equal to about 0.008 $(=\cos\theta)$.

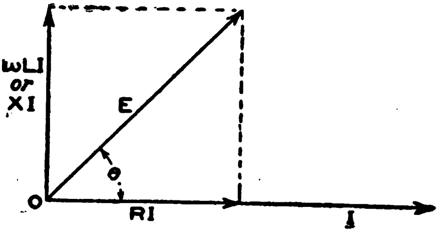

Fig. 52. Diagram of E. M. F.s and Current Relations for Conditions in Fig. 51

One never encounters in practice a circuit in which the reactance is so large compared to the resistance as in the above example; that is, one never encounters one in which the power factor is so small as 0.008. Cases are often met with, however, where the reactance is from two to ten times as large as the resistance. Thus, one of the primary windings of a certain 110-volt induction motor has a resistance of 0.7 ohm and a reactance of 4.2 ohms. With zero load this circuit, equation (23), takes $\frac{110}{\sqrt{0.7^2+4.2^2}} = \frac{110}{4.258}$, or 25.83 amperes; the angle of phase lag of the current, according to equation (24), is about 80½°; and the power factor of the circuit is 0.164.

A circuit which contains a coil wound on an iron core takes more power than is expended in the mere heating of the wire, namely, I^2R, for some power is consumed in the iron core on account of magnetic hysteresis and eddy currents. In the above examples this consumption of power in an iron core is neglected for the sake of simplicity.

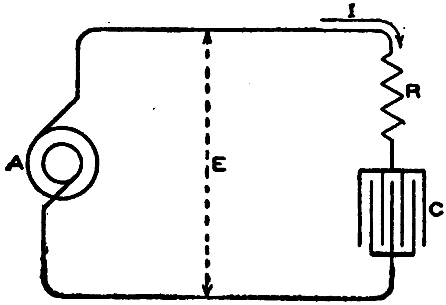

Fig. 53. Circuit Containing Resistance and Capacity

The term *equivalent resistance* is used to designate a fictitious resistance which when multiplied by I^2 gives the actual power consumed by such a circuit, including both the power consumed in heating the wire, and that consumed by core loss. The equivalent resistance of such circuits has a value larger than the mere resistance of the copper winding.

Case C. *Circuits containing resistance and a condenser.* In this case the reactance X $(= -\frac{1}{\omega C})$ is negative, and the current leads the electromotive force in phase, as before pointed out. The tangent of the angle of lead is equal to $\frac{X}{R}$, according to equation (24). Therefore, the angle of lead of the current is small when X is small compared with R, that is, when ωC is large; while the angle of lead of the current approaches 90° when X is large compared with R, that is, when ωC is small.

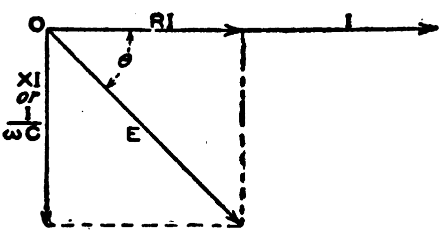

Fig. 54. Diagram of E. M. F. and Current Relations for Conditions of Fig. 53

Fig. 53 represents a circuit containing resistance and a condenser connected to an alternator A; and Fig. 54 shows the relation between the electromotive force and current.

Example. A condenser with a capacity of 2 microfarads (which is large, as condensers go) is connected to alternating-current mains through a resistance coil of 200 ohms. The effective electromotive force between the mains is 1,000 volts, and the frequency is 133 cycles per second, so that the factor ω is equal to $2\pi \times 133$ or 835.7 radians per second; the reactance of the condenser is $\frac{10^6}{2 \times 835.7} = 598.3$ ohms (negative); the impedance of the circuit is $\sqrt{\overline{200}^2 + \overline{598.3}^2} = 630.85$ ohms; the current, according to equation (23), is 1.585 amperes; the current, according to equation (24), is 71° 31′ ahead of the electromotive force in phase; and the power delivered to the circuit is 1,000 volts × 1.585 amperes × cos 71° 31′, which is equal to 502.5 watts ($= I^2R$).

If the above condenser is connected to the 1,000-volt, 133-cycle mains through a wire of negligible resistance, then the current will be $\frac{1{,}000 \text{ volts}}{598.3 \text{ ohms}}$, or 1.671 amperes; the current will be very nearly 90° ahead of the electromotive force in phase; the power factor $\cos \theta$, will be nearly zero; and of course the power delivered to the condenser will be nearly zero.

A circuit containing a condenser takes a little more power than is expended in the mere heating of the wire, namely, I^2R, for some power is consumed in the insulating material between the condenser plates. This power consumed in the dielectric is said to be due to *dielectric hysteresis*. In the above examples, this consumption of power in the insulating material of a condenser is neglected for the sake of simplicity.

Case D. *Circuit in which the inductance reactance ωL is balanced by the capacity reactance $\frac{1}{\omega C}$.* In this case equation (21) reduces to $I = \frac{E}{R}$; that is, the electromotive force acting upon the circuit has to overcome resistance only, as in the case of the non-reactive circuit. This case in which $\omega L - \frac{1}{\omega C}$ is equal to zero is considered again in the following article on resonance.

Electrical Resonance. Consider a circuit, like the one shown in Fig. 45, containing a given resistance R, a given induction L, and a given capacity C. Suppose that the alternator A is at first run at very slow speed so as to give very low frequency, and is then gradually increased in speed so as to cause the frequency to increase. This gradual increase of frequency will cause a gradual increase in the value of the factor ω (equal to 2π times the frequency); and as ω increases, the following relations between inductance reactance ωL and capacity reactance $\frac{1}{\omega C}$ will obtain:

(*a*) At first, when the frequency is very low (few cycles per second), the value of ω is small. Therefore, the inductance reactance ωL is small; the capacity reactance $\frac{1}{\omega C}$ is large; and the total net reactance $\omega L - \frac{1}{\omega C}$ is negative, and very nearly the same as $\frac{1}{\omega C}$ alone.

(*b*) As the frequency increases, the value of ω increases. Therefore, the inductance reactance ωL increases; the capacity reactance $\frac{1}{\omega C}$ decreases; and the total net reactance $\omega L - \frac{1}{\omega C}$ increases in value on account of the increase of ωL, and also on account of the decrease of $\frac{1}{\omega C}$. For a certain critical value of the frequency, ωL becomes equal to $\frac{1}{\omega C}$, so that the total net reactance is then zero. That is

$$\omega L - \frac{1}{\omega C} = 0$$

or

$$\omega L = \frac{1}{\omega C}$$

or

$$\omega^2 = \frac{1}{LC}$$

or

$$\omega = \frac{1}{\sqrt{LC}}$$

or, since ω equals $2\pi f'$, we have

$$f' = \frac{1}{2\pi\sqrt{LC}} \qquad (26)$$

in which f' is the critical value of the frequency for which inductance reactance is balanced by capacity in the given circuit, L being the inductance of the circuit and C the capacity of the condenser. See Fig. 45.

(*c*) As the frequency increases beyond the critical value f', the inductance reactance ωL continues to increase; the capacity reactance continues to decrease; and the net reactance $\omega L - \frac{1}{\omega C}$, now positive in value, continues to increase in value.

Now, imagine the electromotive force of the alternator to be *constant in value*, although increasing in frequency as the alternator is speeded up.* The current in the circuit will at first increase with increasing frequency until the critical frequency f', equation (26), is reached, and then the current will decrease in value as the frequency increases, a maximum value of current being produced at the critical frequency f'. This production of a maximum current at the critical frequency f' is called *electrical resonance*. At critical frequency the reactance $\omega L - \frac{1}{\omega C}$ is zero; and the general equation (21) reduces to $I = \frac{E}{R}$, as explained in Case D, page 49. That is, *the value of the current at the critical frequency is determined solely by the resistance of the circuit and for a given resistance has its maximum value.*

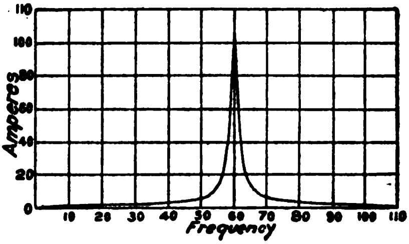

Fig. 55. Graphical Relation of Current and Frequency; E. M. F. Constant

The variation of current in a circuit like that shown in Fig. 45, with increasing frequency, electromotive force being kept constant in value, is shown graphically in Fig. 55, which is calculated from the following data: $E=200$ volts (effective); $R=2$ ohms; $L=0.352$ henry; and $C=20$ microfarads.

The critical frequency corresponding to these values of L and C is 60 cycles per second, according to equation (26). The maximum point of the curve is not a cusp, as would appear from the figure; but the curve is rounded at the top, the figure being drawn on too small a scale to show it.

It should be remarked that the conditions for complete resonance can be obtained only when C and L are constant and concentrated (not distributed), and when the electromotive force is harmonic. If the electromotive force is non-harmonic, it means that it is composed of a number of sine waves having different frequencies. It is evident, therefore, that ωL can never be made exactly equal to the fraction $\frac{1}{\omega C}$ unless there is but one fundamental frequency equal to $f=\frac{\omega}{2\pi}$.

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*In practice this condition could be realised by adjusting the field rheostat of the alternator, or of the exciter, so as to reduce the exciting current as the speed of the alternator is increased.

Multiplication of E. M. F. by Resonance. When resonance exists in a circuit containing an inductance and a condenser in series, the alternating electromotive force ωLI between the terminals of the inductance, and the alternating electromotive force $\frac{I}{\omega C}$ between the terminals of the condenser, may each be much greater than the alternating electromotive force RI which acts upon the circuit. This fact is easily understood by means of the mechanical analogue. If even a very weak periodic force act upon a weight which is suspended from a spiral spring, the weight will be set into violent vibration, provided the frequency of the force is the same as the proper frequency of oscillation of the body. The forces acting on the spring may reach enormously greater values than the periodic force which maintains the motion of the system. Moreover, the forces which act upon the weight to produce its up-and-down acceleration may reach values very much larger than the periodic force which maintains the motion.

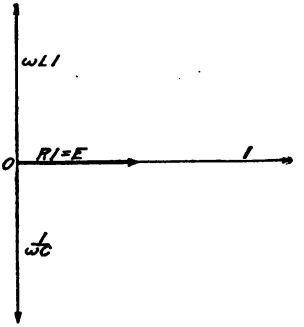

Fig. 56. Diagram Showing Multiplication of E. M. F.s by Resonance

Example. A coil having an inductance of 0.352 henry and a resistance of 2 ohms, and a condenser of 20 microfarads capacity, are connected in series between alternating-current mains. The critical frequency of this circuit is 60 cycles per second, according to equation (26). The electromotive force between the mains is 200 volts, and its frequency is 60 cycles per second. The current in the circuit is $\frac{200 \text{ volts}}{2 \text{ ohms}}$, or 100 amperes, according to equation (21); the effective electromotive force between the condenser terminals is 13,270 volts effective $(=\frac{I}{\omega C})$; and the electromotive force between the terminals of the inductance is also 13,270 volts effective $(=\omega LI)$.

The multiplication of electromotive force by resonance may be clearly understood with the help of the clock diagram, Fig. 56.

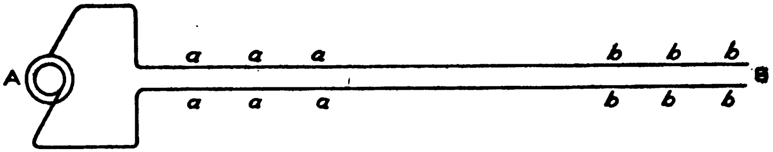

Fig. 57. An Alternator Delivering Current to a Long Transmission Line

The electromotive force ωLI required to overcome inductance reactance is equal and opposite to the electromotive force $\frac{I}{\omega C}$ required to overcome capacity reactance, as shown in Fig. 56, so that the geometric sum of ωLI, $\frac{I}{\omega C}$, and RI is, simply, RI. A transmission line has both inductance and capacity and, therefore, electrical resonance may occur on a transmission line. The phenomena of a transmission line, however, are very greatly complicated by the fact that the capacity is distributed; and a simple explanation of line reasonance can be given only by approximation.

For example, an alternator A, Fig. 57, delivers current to a long transmission line. The resonance effects are nearly independent

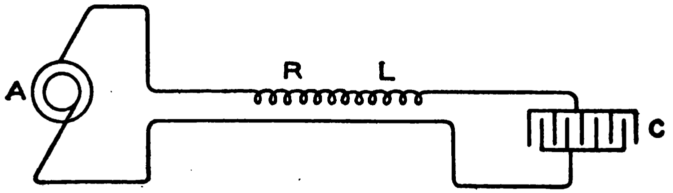

Fig. 58. Diagram of Conditions Shown in Fig. 57 with Transmission Lines Insulated from Each Other

of whether the receiving apparatus at the end of the line, viz, at B, is connected into the circuit or not. We shall, therefore, consider that the receiving apparatus is disconnected and, furthermore, that the ends of the two transmission lines are insulated from each other.

A first approximation to the behavior of the line may be obtained by looking upon the distant end of the line *b b b* as a condenser purely and simply, while the near end of the line *a a a* is looked upon simply as an inductance. The transmission line shown in Fig. 57 is then equivalent to the combination shown in Fig. 58, which is identical with the combination shown in Fig. 45. The value of L may be taken as the inductance of, say, half the length of the line; and the capacity C may be taken as the capacity of the distant half of the transmission line. Then, if the alternator A gives a frequency equal to the critical value of these values of L and C, as per equation (26), we shall have resonance, and the electromotive force between the lines at the distant end (between the terminals of C, Fig. 58) may be greatly in excess of the electromotive force of the alternator A. This condition actually occurs in the practical operation of long transmission lines; and it is not an uncommon thing to have as much as 11,000 volts at the receiving end of a long transmission line when the electromotive force of the generator is but 10,000 volts.

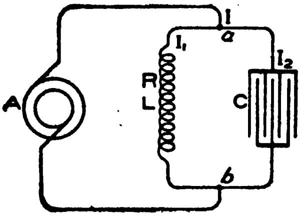

Fig. 59. Circuit Showing Condenser and Inductance in Parallel

Multiplication of Current by Resonance. An alternator A, Fig. 59, delivers current to a circuit which divides at the points a and b into two branches, one branch containing an inductance L and the other branch containing a capacity C, as shown. The two branches constitute a closed circuit in and of themselves; and if the frequency of the alternator is equal to the critical frequency of the circuit constituted by the two branches, that is, if the frequency of the alternator is equal to $\frac{1}{2\pi\sqrt{LC}}$, as per equation (26), then the small current I from the alternator will divide into two currents I_1 and I_2 in the respective branches, and the currents I_1 and I_2 may each be very much larger in value than the undivided cur-

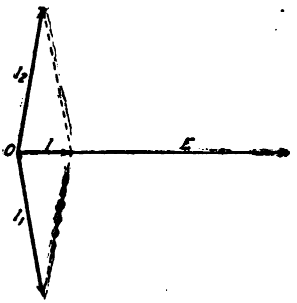

Fig. 60. Vector Diagram for Multiplication of Current by Resonance

rent I. The fact is that, because of resonance, a very large current is made to surge back and forth around the closed circuit formed by the two branches.

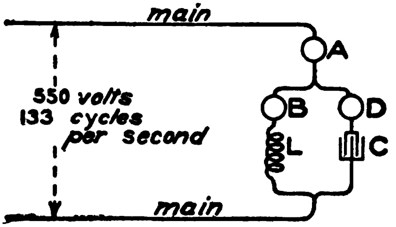

Fig. 61. Diagram of Circuit Showing Multiplication of Current by Resonance

The multiplication of current by resonance may be clearly understood with the help of the clock diagram, Fig. 60. The line OE represents the electromotive force between the branch points and $a\,b$, Fig. 59; the line I_1 represents the lagging current which the electromotive force E produces in the branch containing the inductance; the line I_2 represents the leading current which the electromotive force E produces in the branch containing the condenser; and the line I, which is the geometric sum of I_1 and I_2, represents the total current in the undivided part of the circuit in Fig. 59.

For example, three similar 32-candle-power incandescent lamps A, B, and D, Fig. 61, each having 100 ohms resistance, are connected as shown, to 550-volt mains; L is an inductance of 0.597 henry; and C, a capacity of 2.49 microfarads. Then the current flowing through the lamp A is not quite 0.4 ampere, while one ampere of current flows through each of the lamps B and D.

Miscellaneous Considerations. *Condenser as Compensator for Lagging Current.* An alternator may be designed to develop a certain effective electromotive force E, and to deliver a certain effective current I, at full load. The permissible power output of such an alternator would be EI watts to a non-reactive circuit having unity power factor ($\cos\theta = 1$); but if the receiving circuit is reactive, the permissible power output of the alternator is only $EI\cos\theta$, where the power factor ($\cos\theta$) may in practice have a value of .8 or less. If a condenser C of sufficiently large capacity, Fig. 62, is connected across the terminals of an alternator A in parallel with

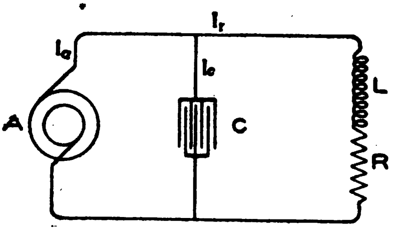

Fig. 62. Diagram of Condenser Compensating for Lagging Current

an inductive receiving circuit *RL*, the effect of *L* will be neutralized; the current delivered by *A* will be in phase with the electromotive force of *A;* and the permissible power output will be *EI*. The condenser is said to *compensate* for the lagging current taken by the inductive receiving circuit.

The compensation produced by a condenser is due to the fact that the alternating current taken by it is ahead of the electromotive force in phase, while that taken by the reactive receiving circuit is behind the electromotive force in phase.

Another advantage, aside from the increase of the permissible power output of the generator, that would result from this compensating of lagging current by means of a condenser, is that the electromotive force of the alternator would not fall off so much with increase of load as is the case when lagging current is not compensated for. The cost, however, of large condensers is so great that their use for compensation of lagging current is not commercially practicable, as may be seen from the following discussion:

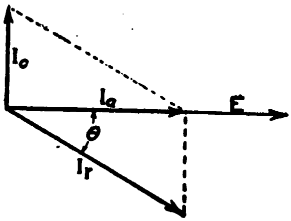

Fig. 63. Vector Diagram of Conditions Shown in Fig. 62

Let I_r be the current delivered to the receiving circuit *RL*, Fig. 62. Let I_c be the current delivered to the condenser *C;* this current is 90 degrees ahead of *E* in phase. Let I_a be the current delivered by the alternator *A*. It is desired that I_a be in phase with *E*, as shown in Fig. 63. Let $\cos\theta$ be the power factor of the receiving circuit *RL*.

From Fig. 63 it is evident that I_c is equal and opposite to that component of I_r which is at right angles to *E*, namely, $I_r \sin\theta$. Therefore

$$I_c = I_r \sin\theta$$

Now, I_c is equal to $\dfrac{E}{\dfrac{1}{\omega C}}$ $(=\omega CE)$, that is, is equal to the electromotive force between the condenser terminals divided by the reactance of the condenser. The value of $\sin\theta$ is $\dfrac{\omega L}{\sqrt{R^2+\omega^2 L^2}}$; and $I_r =$

$\frac{E}{\sqrt{R^2 + \omega^2 L^2}}$; so that, substituting the values of I_r and sin θ, the above equation for I_c becomes

$$\omega C E = \frac{\omega L E}{R^2 + \omega^2 L^2}$$

whence

$$C = \frac{L}{R^2 + \omega^2 L^2} \tag{27}$$

in which R is the resistance (in ohms) of the receiving circuit; L is the inductance (in henrys) of the receiving circuit; ω is a factor equal to 2π times the frequency in cycles per second; and C is the capacity (in farads) of the condenser required to compensate for the lagging current delivered to the receiving circuit. Suppose an alternator having an electromotive force of 1,100 volts and a frequency of 60 cycles per second, delivers 102.4 amperes of current to a receiving circuit of which the power factor is 0.871 (9.35 ohms resistance and 0.014 henry inductance). The capacity of a condenser, which will compensate for the lagging current in this case, may be calculated from equation (27) as follows:

$$C = \frac{0.014}{9.35^2 + (2\pi \times 60)^2 \times 0.014^2}$$

$$= 0.0001214 \text{ farad} = 121.4 \text{ microfarads}$$

This condenser would take from the mains a current of 50.34 amperes, which would be 90 degrees ahead of the e. m. f. in phase, and this current would be equal and opposite to the wattless component of the 102.4 amperes of current delivered to the inductive circuit. Such a condenser would require about 114,000 leaves of tinfoil 8 inches × 10 inches, separated by 114,000 leaves of paraffined paper, each 0.03 inch in thickness. This would give a stack of condenser leaves of about 400 feet total thickness; and the cost of material and labor would be at least $10 per microfarad.

Such a condenser would be impracticable but a synchronous motor with over-excited field magnets behaves like a large condenser in that it takes an armature current which is ahead of the electromotive force in phase. The synchronous motor is often used in practice to compensate for lagging line current and when so operated is called a *rotary condenser*.

Circuits in Series. An alternator A, Fig. 64, delivers current to two coils (or elements) in series as shown. Let R_1 be the resistance and X_1 the reactance of coil *1*. Let R_2 be the resistance and X_2 the reactance of coil *2*. Let I be the current flowing through the circuit; E the electromotive force between the mains; E_1 the electromotive force between the terminals of coil *1*; and E_2 the electromotive force between the terminals of coil *2*. Let θ be the phase difference between E and I; θ_1 the phase difference between E_1 and I; and θ_2 the phase difference between E_2 and I, as shown in Fig. 65.

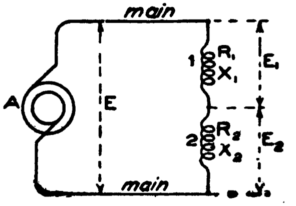

Fig. 64. Two Elements in Series in Alternating Circuit

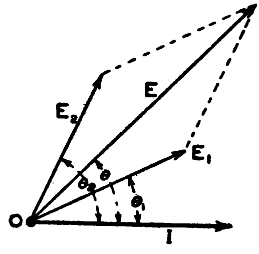

Fig. 65. Diagram of E. M. F. Conditions for Fig. 64

Of course θ_1 is the angle whose tangent is $\frac{X_1}{R_1}$; θ_2 is the angle whose tangent is $\frac{X_2}{R_2}$; and θ is the angle whose tangent is $\frac{(X_1 + X_2)}{(R_1 + R_2)}$, according to equation (24).

Example. Two impedance coils have resistances of 5 and 8 ohms and inductances of 0.01 and 0.2 henry, respectively. If these coils are connected in series across 220-volt, 60-cycle mains, find: (a) the current; (b) the voltages impressed across the coils; and (c) the phase relations between the current and the voltages impressed across the coils.

SOLUTION. We have $R_1 = 5$, $R_2 = 8$, $E = 220$, $f = 60$, $L_1 = 0.01$, $L_2 = 0.2$. Then $\omega = 2\pi f = 2\pi \times 60 = 377$ radians per second. The reactance of coil 1 is $X_1 = \omega L_1 = 377 \times 0.01 = 3.77$ ohms, and the reactance of coil 2 is $X_2 = \omega L_2 = 377 \times 0.2 = 75.4$ ohms. The total impedance of the circuit is $Z = \sqrt{(R_1 + R_2)^2 + (X_1 + X_2)^2} = \sqrt{(5+8)^2 + (3.77 + 75.4)^2} = 80.23$ ohms, so that the current is

$$\text{(a)} \qquad I = \frac{E}{Z} = \frac{220}{80.23} = 2.74 \text{ amperes}$$

(b) *To find the voltages E_1 and E_2.* (See Figs. 64 and 65.) The impedance of coil *1* is $Z_1 = \sqrt{5^2 + 3.77^2} = 6.262$ ohms and the impedance of coil *2* is $Z_2 = \sqrt{8^2 + 75.4^2} = 75.82$ ohms. The magnitude of the voltage E_1 is

$$E_1 = IZ_1 = 2.74 \times 6.262 = 17.16 \text{ volts},$$

and the magnitude of the voltage E_2 is

$$E_2 = IZ_2 = 2.74 \times 75.82 = 207.7 \text{ volts}$$

(c) The phase angle θ_1 between the current and E_1 is obtained from the relation $\tan \theta_1 = \frac{X_1}{R_1} = \frac{3.77}{5} = 0.754$, whence

$$\theta_1 = 37^\circ\ 1'$$

Similarly the phase angle θ_2 between I and E_2 is found to be $\tan \theta_2 = \frac{75.4}{8} = 9.425$, or

$$\theta_2 = 83^\circ\ 56'$$

and $\tan \theta = \frac{X_1 + X_2}{R_1 + R_2} = \frac{79.17}{13} = 6.090$, whence

$$\theta = 80^\circ\ 41'$$

Referring to Fig. 65 it is evident that by taking the line (or vector) representing the current I as the axis of reference, the lines representing the electromotive forces, E_1, E_2, and E may each be resolved into two components, one parallel to or in phase with I and the other perpendicular to or in quadrature with I. Then the sum of the components of E_1 and E_2 parallel to I will be equal to the component of E parallel to I, and similarly the sum of the components of E_1 and E_2 perpendicular to I, will be equal to the vertical component of E. Expressing these statements in equation form and substituting the values, we have the following:

$$\begin{array}{ccccc} (E_1 \cos \theta_1) & + & (E_2 \cos \theta_2) & = & (E \cos \theta) \\ (17.16 \cos 37^\circ\ 1') & + & (207.7 \cos 83^\circ\ 56') & = & (220 \cos 80^\circ\ 40.3') \\ (13.71) & + & (21.98) & = & (35.69 \text{ approx.}) \end{array}$$

and

$$\begin{array}{ccccc} (E_1 \sin \theta_1) & + & (E_2 \sin \theta_2) & = & (E \sin \theta) \\ (17.16 \sin 37^\circ\ 1') & + & (207.7 \sin 83^\circ\ 56') & = & (220 \sin 80^\circ\ 40.5') \\ (10.34) & + & (206.7) & = & (217 \text{ approx.}) \end{array}$$

Circuits in Parallel. An alternator A, Fig. 66, delivers current to two coils (or elements) in parallel as shown.

Let R_1 be the resistance, and X_1 the reactance of coil *1;* let R_2 be the resistance, and X_2 the reactance of coil *2;* let E be the electromotive force between the coil terminals; let I_1 be the current in coil *1*, I_2 the current in coil *2*, and I the total current; and let θ be the angle of phase difference between E and I; θ_1 the phase angle between I_1 and E; and θ_2 the angle between I_2 and E, as shown in Fig. 67.

The angles θ_1, θ_2, are known from the relations

$$\tan\theta_1=\frac{X_1}{R_1} \qquad \tan\theta_2=\frac{X_2}{R_2}$$

The branch currents are: $I_1=\dfrac{E}{\sqrt{R_1^2+X_1^2}}$ and $I_2=\dfrac{E}{\sqrt{R_2^2+X_2^2}}$ but since they are not in phase, they cannot be added algebraically to obtain the resultant current I, but must be added geometrically as is done in Fig. 67. By trigonometry the magnitude of I is

$$I=\sqrt{I_1^2+I_2^2+2I_1\,I_2\cos(\theta_2-\theta_1)}$$

In the case of circuits in parallel the voltage across each branch is the same, and it is, therefore, convenient to determine all currents by their magnitudes and their relations with respect to this common voltage. Thus in Fig. 67 draw the line OE to represent the voltage in magnitude; it is convenient to draw it horizontal since the current vectors in the clock diagram are to be referred to this vector voltage as the axis of reference. The vectors OI_1 and OI_2 are then drawn from O to represent the currents I_1 and I_2 in magnitude and with the proper directions as determined by the phase angles θ_1 and θ_2.

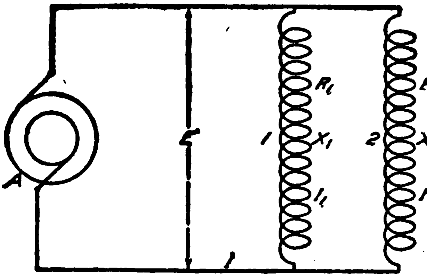

Fig. 66. Two Elements in Parallel in Alternating Circuit

The magnitude and phase of the resultant current I may be obtained graphically from the clock diagram, but this method is not to be recommended where accuracy is sought.

To obtain by calculation the phase angle between the main current I and the voltage E, it is necessary to resolve I into its components parallel to and in quadrature with E.

The active, or power, component of the current I_1 is

$$I_1 \cos \theta_1 = \frac{R_1 I_1}{Z_1} = \frac{E}{Z_1} \times \frac{R_1}{Z_1} = E g_1$$

where $g_1 = \frac{R_1}{Z_1^2}$ is called the *conductance* of circuit 1.

Similarly the power component of I_2 is

$$I_2 \cos \theta_2 = \frac{R_2 I_2}{Z_2} = E g_2$$

The "reactive" or "wattless" component of the current I_1 is

$$I_1 \sin \theta_1 = I_1 \frac{X_1}{Z_1} = \frac{E}{Z_1} \times \frac{X_1}{Z_1} = E b_1$$

where $b_1 = \frac{X_1}{Z_1^2}$ is called the *susceptance* of circuit 1.

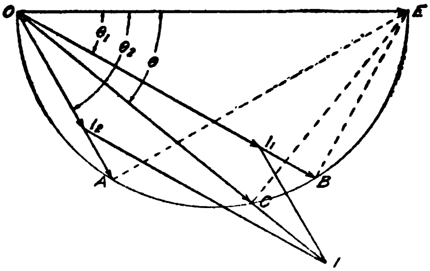

Fig. 67. Vector Diagram of Current Conditions for Fig. 66

Similarly the reactive component of the current I_2 is

$$I_2 \sin \theta_2 = I_2 \frac{X_2}{Z_2} = \frac{E}{Z_2} = E b_2$$

Since I is the *vector sum* of the two branch currents

$$I = I_1 + I_2$$

The power component of the resultant current is the sum of the active components of the two branch currents, or

$$I \cos \theta = I_1 \cos \theta_1 + I_2 \cos \theta_2 = E g_1 + E g_2 = E\,(g_1 + g_2)$$

and the reactive component of I is

$$I \sin \theta = I_1 \sin \theta_1 + I_2 \sin \theta_2 = E b_1 + E b_2 = E\,(b_1 + b_2)$$

Therefore

$$\tan\theta=\frac{I\sin\theta}{I\cos\theta}=\frac{b_1+b_2}{g_1+g_2}$$

and the magnitude of the vector I being the hypotenuse of a right-angle triangle, having a base $I\cos\theta$, and an altitude $I\sin\theta$, is

$$I=E\sqrt{(g_1+g_2)^2+(b_1+b_2)^2}$$

By drawing a semicircle on OE as a diameter and extending the current vectors I_1, I_2, and I, till they meet the semicircle at B, A, and C in Fig. 67, some important relations may be deduced. Join the points A, B, and C with E, thus completing the triangles OAE, OBE, and OCE. They are all right-angle triangles because inscribed in semicircle.

A careful study of Fig. 67 will show that $OA=R_2I_2$, $AE=X_2I_2$, (see page 43), and that $\sqrt{\overline{OA}^2+\overline{AE}^2}=OE=E$ from which $R_2=\frac{OA}{I_2}$, $X_2=\frac{AE}{I_2}$, and $Z_2=\frac{OE}{I_2}$.

Similarly $OB=R_1I_1$, $BE=X_1I_1$, and $\sqrt{\overline{OB}^2+\overline{BE}^2}=OE=E$ and $RI=OC$, $CE=XI$, and $\sqrt{\overline{OC}^2+\overline{CE}^2}=E$ in which R is called the "equivalent resistance" of the total circuit, and X its "equivalent reactance." In other words, if the branched circuits were replaced by a single coil having a resistance equal to R and a reactance equal to X, the coil would take the same current from the mains as before, with the same angle of lag, and would absorb the same total power. The impedance of this single "equivalent coil" is not the sum of the impedances of the separate coils, but is $\frac{E}{I}$.

Example. An impedance coil having a resistance of 19.05 ohms and a reactance (at a frequency of 60 cycles) of 11 ohms is connected in parallel with a second impedance coil having a resistance of 22 ohms and a reactance of 38.1 ohms across 220-volt, 60-cycle mains. Find (a) the current in each coil and the total (or line) current; (b) the phase relation of the currents; (c) the power factor of the entire circuit and the power expended in it; and (d) the "equivalent" resistance and reactance of the circuit.

Solution. (a) To find the currents, we have

$$I_1 = \frac{E}{\sqrt{R_1^2 + X_1^2}} = \frac{220}{\sqrt{(19.05)^2 + (11)^2}} = \frac{220}{22} = 10 \text{ amperes}$$

and

$$I_2 = \frac{E}{\sqrt{R_2^2 + X_2^2}} = \frac{220}{\sqrt{(22)^2 + (38.1)^2}} = \frac{220}{44} = 5 \text{ amperes}$$

The above values are simply the *magnitudes* of the branch currents. Before they can be represented in a clock diagram, their directions must be found.

(b) $\tan \theta_1 = \frac{X_1}{R_1} = \frac{11}{19.05} = 0.577$, or

$$\theta_1 = -30^\circ$$

The minus sign means a lagging current.

$$\tan \theta_2 = \frac{X_2}{R_2} = \frac{38.1}{22} = 1.732, \text{ or}$$

$$\theta_2 = -60^\circ$$

The angle of phase difference between I_1 and I_2 is

$$\theta_2 - \theta_1 = -60^\circ - (-30)^\circ = -30^\circ$$

The vector currents may now be drawn to scale as is done in Fig. 67. The resultant or vector sum of the currents I_1 and I_2 may be found by drawing the diagonal of the parallelogram on I_1 and I_2 as sides. The main current is then OI.

Its magnitude may be found from equation (28)

$$I = \sqrt{I_1^2 + I_2^2 + 2I_1 I_2 \cos(\theta_2 - \theta_1)}$$

$$= \sqrt{10^2 + 5^2 + (2 \times 10 \times 5 \times 0.866)} = \sqrt{211.6} = 14.55 \text{ amperes.}$$

To find the phase of I, we have

$$\tan \theta = \frac{b_1 + b_2}{g_1 + g_2}$$

where $b_1 = \frac{X_1}{Z_1^2} = \frac{11}{19.05^2 + 11^2} = \frac{11}{22^2} = 0.0227$; $b_2 = \frac{X_2}{Z_2^2} = \frac{38.1}{(44)^2} = 0.01968$;

$g_1 = \frac{R_1}{Z_1^2} = \frac{19.05}{(22)^2} = 0.0394$; and $g_2 = \frac{R_2}{Z_2^2} = \frac{22}{(44)^2} = 0.01136$.

$$\tan\theta = \frac{0.0227+0.01968}{0.0394+0.0114} = \frac{0.04238}{0.0508} = 0.8359$$

or

$$\theta = 39° \ 53'$$

which is the angle by which I lags behind E.

(c) The power factor of coil *1* is $\cos\theta_1$—$\cos 30° = 0.86603$ and the power expended in it is $I_1^2 R_1 = 10^2 \times 19.05 = 1905$ watts, or it is $E^2 g_1 = (220)^2 \times 0.0394 = 1905$ watts.

The power factor of coil *2* is $\cos\theta_2 = \cos 60° = 0.5$ and the power expended in it is $I_2^2 R_2 = (5)^2 \times 22 = 550$ watts, or it is $E^2 g_2 = (220)^2 \times 0.01136 = 550$ watts.

The power expended in coils *1* and *2* $= 1905 + 550 = 2455$ watts.

The power factor of the entire circuit (two branches) is

$$\cos\theta = \cos 39° \ 53' = 0.76735$$

and the power expended in it is

$$EI\cos\theta = 220 \times 14.55 \times 0.76735 = 2455 \text{ watts}$$

(d) The "equivalent resistance" R is

$$\frac{OC}{I} = \frac{220 \cos 39° \ 53'}{14.55} = \frac{168.81}{14.55} = 11.6 \text{ ohms}$$

The power absorbed in R is

$$I^2R = (14.55)^2 \times 11.6 = 2455 \text{ watts}$$

The "equivalent reactance" X is

$$\frac{CE}{I} = \frac{220 \sin 39° \ 53'}{14.55} = 9.7 \text{ ohms}$$

The current I lags behind the voltage E, by an angle θ given by $\tan\theta = \frac{X}{R}$ or $\tan\theta = \frac{9.7}{11.6} = 0.836$ from which $\theta = -39° \ 53'$ the same as found above.

Electromotive Force Losses in Transmission Lines. An alternator of which the electromotive force is E, delivers current over a transmission line of which the resistance is R_1 and the reactance (inductance reactance) is X_1, to a receiving circuit of which the resistance is R_2 and the reactance is X_2. The total electromotive force used to overcome the resistance and the reactance of the transmission line is E_1, and the electromotive force between the terminals of the receiving circuit is E_2. The current delivered is I. Then the general relation between E, E_1, and E_2 is as shown in Fig. 65, *except that E_2 is usually very much larger than E_1 in value.* There are three interesting and simple special cases of electromotive force losses in transmission lines, as follows:*

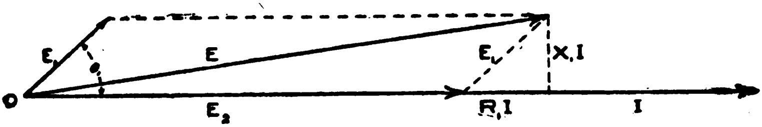

Fig. 68. Diagram of E. M. F. Losses in Transmission Lines—Receiving Circuit Non-Reactive

CASE 1. *When the receiving circuit is non-reactive.* In this case the electromotive force E_2 between the terminals of the receiving circuit is in phase with I, the power factor of the receiving circuit is unity, and the general diagram of Fig. 65 takes the form shown in Fig. 68. The total electromotive force E_1 consumed in the line is sometimes called the *impedance loss* or *drop* and its two components R_1I and X_1I, as shown in Fig. 68, are called the *resistance loss* and the *reactance loss*, respectively. Now, a careful inspection of Fig. 68 makes it evident that the numerical difference between the values of E and E_2 is very nearly equal to the resistance loss in the line R_1I; and that the reactance loss in the line X_1I has little to do with the difference between the values of E and E_2. Therefore, *when the receiving circuit is non-reactive, the difference in value between generator electromotive force E and receiver electromotive force E_2 is sensibly equal to the resistance loss of electromotive force in the line, and sensibly independent of the reactance loss of electromotive force in the line.*

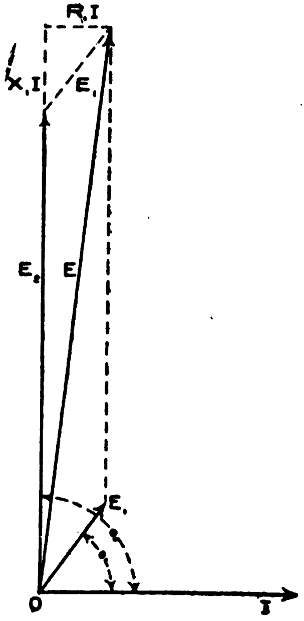

Fig. 69. E. M. F. Losses When the Receiving Circuit is Highly Reactive

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*This discussion applies to comparatively short lines, ten miles or less in length, inasmuch as the capacity of the line is not here taken into account.

CASE 2. *When the receiving circuit is highly reactive.* In this case the electromotive force E_2 between the receiving circuit terminals is nearly 90 degrees ahead of I in phase, and the general diagram of Fig. 65 takes the form shown in Fig. 69. A careful inspection of Fig. 69 makes it evident that the difference in value of E and E_2 is very nearly equal to the reactance loss in the line X_1I; and that the resistance loss in the line R_1I has little to do with the difference between the values of E and E_2. Therefore, *when the receiving circuit is highly reactive, the numerical difference in value between generator electromotive force E and receiver electromotive force E_2 is sensibly equal to the reactance loss of electromotive force in the line, and sensibly independent of the resistance loss of electromotive force in the line.*

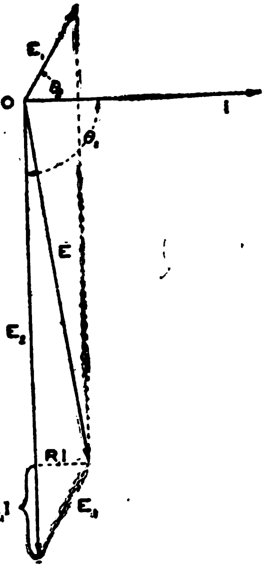

Fig. 70. E. M. F. Losses with Large Capacity Reactance

CASE 3. *When the receiving circuit has large capacity reactance.* In this case the electromotive force E_2 between the receiving circuit terminals is nearly 90 degrees behind I in phase, and the general diagram of Fig. 65 takes the form shown in Fig. 70. A careful inspection of Fig. 70 makes it evident that the difference between the values of E and E_2 is very nearly equal to the reactance loss in the line X_1I, *and that this reactance loss is added to the generator electromotive force E to give the receiver electromotive force E_2.* Inspection of Fig. 70 shows furthermore that the resistance loss in the line R_1I has little to do with the difference in value of E and E_2. Therefore, *when the receiving circuit has a high capacity reactance, the reactance loss in the line is sensibly equal to the rise in value of the electromotive force between generator and receiver, and this rise in value is sensibly independent of the resistance loss of electromotive force in the line.* It is somewhat confusing to speak of the electromotive force X_1I as reactance *loss* when the receiving circuit has a high capacity reactance; it would be better in this case to speak of X_1I as the reactance *gain* of electromotive force in the line.

Examples. 1. Assume E_2 equals 2000 volts, Z_2 equals 20 ohms, R_1 equals 1 ohm, and X_1 equals 4 ohms. Calculate E for Cases 1, 2, and 3.

SOLUTION. Case 1 (Fig. 68).

$$R_2=Z_2,\ I=\frac{E_2}{R_2}=100 \text{ amperes}$$

$$R_1I=100 \text{ volts} \qquad X_1I=400 \text{ volts}$$

$$E=\sqrt{(E_2+R_1I)^2+(X_1I)^2}=2138 \text{ volts}$$

Case 2 (Fig. 69).

$$X_2=Z_2\ I=\frac{E_2}{X_2}=100 \text{ amperes (lagging)}$$

$$R_1I=100 \text{ volts} \qquad X_1I=400 \text{ volts}$$

$$E=\sqrt{(E_2+X_1I)^2+(R_1I)^2}=2402 \text{ volts}$$

Case 3 (Fig. 70).

$$X_2=Z_2 \qquad I=\frac{E_2}{X_2}=100 \text{ amperes (leading)}$$

$$R_1I=100 \text{ volts} \qquad X_1I=400 \text{ volts}$$

$$E=\sqrt{(E_2-X_1I)^2+(R_1I)^2}=1603 \text{ volts}$$

2. Given E_2 equals 2000 volts, R_2 equals 16 ohms, X_2 equals 12 ohms, R_1 equals 1 ohm, and X_1 equals 4 ohms. Find E, when (*a*) X_2 is an inductive reactance; and when (*b*) X_2 is a capacity reactance.

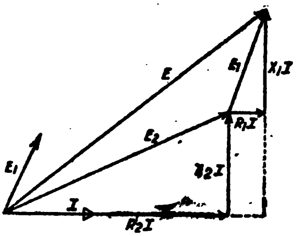

Fig. 71. Vector Diagram, for Example 2, "a"

SOLUTION. (*a*)

$$I=\frac{E_2}{\sqrt{R_2^2+X_2^2}}=100 \text{ amperes}$$

From Fig. 71

$$E=\sqrt{(R_2I+R_1I)^2+(X_2I+X_1I)^2}$$

$$=\sqrt{(1600+100)^2+(1200-400)^2}=2335 \text{ volts}$$

(*b*) As before

$$I=100 \text{ amperes}$$

From Fig. 72

$$E=\sqrt{(R_2I-R_1I)^2+(X_1I-X_2I)^2}$$

$$=\sqrt{(1600+100)^2+(1200-400)^2}=1879 \text{ volts}$$

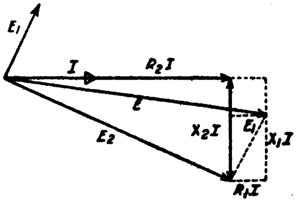

Fig. 72. Vector Diagram for Example 2, "b"

3. Given E equals 2200 volts, R_2 equals 16 ohms, X_2 equals 12 ohms, R_1 equals 1 ohm, and X_1 equals 4 ohms. Find E_2, when (*a*) X_2 is an inductive reactance; and when (*b*) X_2 is a capacity reactance.

SOLUTION. (*a*)

$$I=\frac{E}{\sqrt{(R_2+R_1)^2+(X_2-X_1)^2}}=94.44 \text{ amperes}$$

$$E_2=\sqrt{(R_2^2+X_2^2)}I=20\times 94.44=1889 \text{ volts}$$

(*b*)

$$I=\frac{E}{\sqrt{(R_2+R_1)^2+(X_1-X_2)^2}}=117.15 \text{ amperes}$$

$$E_2=\sqrt{R_2^2+X_2^2}\ I=2343 \text{ volts}$$

Electromotive Force Losses in Alternator Armatures. Let E be the total induced electromotive force in the armature of an alternator. A portion E_1 of this electromotive force is used to overcome the resistance R_1 and the reactance X_1 of the armature; and the remainder E_2 is available at the terminals of the alternator for producing current in the outside circuit, of which the resistance is R_2 and the reactance is X_2. The general relation between E, E_1, and E_2 is as shown in Fig. 65, *except that* E_2 *is usually very much larger than* E_1 *in value.* There are three interesting and simple special cases of electromotive force losses in alternator armatures, as follows:

CASE 1. *When the receiving circuit is non-reactive.* In this case the electromotive force E_2 between the terminals of the alternator is in phase with the current I delivered by the machine, and the general diagram of Fig. 65 takes the form shown in Fig. 68, from which it is evident that when the receiving circuit is non-reactive, the numerical difference in value between the total induced electromotive force E and the terminal electromotive force E_2 of the machine is sensibly equal to the resistance loss of electromotive force R_1I in the armature, and sensibly independent of the reactance loss of electromotive force X_1I in the armature.

CASE 2. *When the receiving circuit is highly reactive.* In this case the electromotive force E_2 between the terminals of the alternator is nearly 90 degrees ahead of I in phase, and the general diagram of Fig. 65 takes the form shown in Fig. 69, from which it is evident that when the receiving circuit is highly reactive the numerical difference in value between the total induced electromotive force E and the terminal electromotive force E_2 of the machine is sensibly equal to the reactance loss of electromotive force X_1I in the armature, and sensibly independent of the resistance loss of electromotive force R_1I in the armature.

CASE 3. *When the receiving circuit has large capacity reactance.* In this case the electromotive force E_2 between the terminals of the alternator is nearly 90 degrees behind I in phase, and the general diagram of Fig. 65, takes the form shown in Fig. 70, from which it is evident that when the receiving circuit has a high capacity reactance the difference in value between E and E_2 is sensibly equal to X_1I, E_2 *being larger than* E, and sensibly independent of the resistance loss of electromotive force R_1I in the armature.

EXAMINATION PAPER

ALTERNATING-CURRENT MACHINERY

PART I

Read Carefully: Place your name and full address at the head of the paper. Any cheap, light paper like the sample previously sent you may be used. Do not crowd your work, but arrange it neatly and legibly. *Do not copy the answers from the Instruction Paper; use your own words, so that we may be sure you understand the subject.*

---

1. What is an alternator?

Define *cycle, frequency,* and *period.*

A.C. Machinery I
Questions Nos. 1, 12, 16, 19 and 20 are optional and may be omitted if the student is limited in the matter of time. The omission will not affect his grade as the principles are covered in the other problems in the examination.

phase 30 degrees,
Select two instantaneous values and add these by a vector diagram.

9. The maximum value of a certain e.m.f. is 311.124 volts. What is the effective value?

10. Calculate the inductance of a coil containing 200 turns of wire wound in a thin layer on a long wooden cylinder of 12-inch length and $\frac{3}{4}$-inch mean radius.

ALTERNATING-CURRENT MACHINERY

PART I

Read Carefully: Place your name and full address at the head of the paper. Any cheap, light paper like the sample previously sent you may be used. Do not crowd your work, but arrange it neatly and legibly. *Do not copy the answers from the Instruction Paper; use your own words, so that we may be sure you understand the subject.*

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1. What is an alternator?

2. Define *cycle*, *frequency*, and *period*.

3. If an alternator has 8 poles and runs at a speed of 900 r.p.m., what is its frequency?

4. State the effects on the current, the resistance, the diameter, and the weight of the wire resulting from the transmission of a certain amount of power for a given per cent of loss by raising the voltage at the receiving end four times.

5. State the Power Law and Ohm's Law for alternating-current circuits.

6. What is meant by inductance and capacity?

7. Give definitions of *maximum*, *effective*, and *average values*, as used in connection with alternating currents.

8. Draw a curve of two harmonic e.m.f.'s which differ in phase 30 degrees, and add them by means of a third curve. Select two instantaneous values and add these by a vector diagram.

9. The maximum value of a certain e.m.f. is 311.124 volts. What is the effective value?

10. Calculate the inductance of a coil containing 200 turns of wire wound in a thin layer on a long wooden cylinder of 12-inch length and $\frac{3}{4}$-inch mean radius.

11. Three capacities of 1.6, 0.8, and 1.36 microfarads are connected in series. What is their joint capacity?

12. By what symbols is reactance expressed? Give the meaning of each symbol.

13. Point out the differences between resistance, reactance, and impedance.

14. A non-inductive resistance takes 5 amperes from a 110-volt 133-cycle circuit. What current will it take from a 220-volt 60-cycle circuit?

15. A condenser with the capacity of 1.17 microfarads is connected to alternating-current mains through a resistance of 60 ohms. The e.m.f. between the mains is 660 volts and the frequency 60 cycles. Calculate the current and the power factor.

16. What is the critical value of the frequency?

17. How can a condenser compensate for lagging current?

18. Give the values of conductance and susceptance in symbols, stating the meaning of each symbol.

19. What is an exciter? Explain its function.

20. What is the form factor of a circuit and what is its least possible value?

21. Define *synchronism* of alternating e.m.f. or currents.

22. Define the following terms: steady direct current; pulsating direct current; periodic alternating current.

23. The inductances of the impedance coils *1* and *2* in Fig. 64 are 0.008 and 1.5 henries, and the resistances are 4 and 6 ohms, respectively. If E is 120 volts and the frequency is 25 cycles, find (a) the current I; (b) E_1 and E_2.

24. Draw vector diagram as in Fig. 65 showing phase relations between I and E, E_1, and E_2 for problem 23.

25–26–27. Solve the example at the bottom of Page 63, when the frequency is 25 cycles, all other conditions remaining the same.

After completing the work, add and sign the following statement:
I hereby certify that the above work is entirely my own.
(Signed)

JAN 19 1921